Quantum Mechanics 3, Spring 2012 CMI

Problem set 10

Due by beginning of class on Monday Mar 26, 2012

Klein-Gordon equation and Dirac matrices

1. $\langle 10 \rangle$ We try the change of wave function $\psi(r,t) = e^{imc^2 t/\hbar}\chi(r,t)$ in the Klein-Gordon equation where we have in mind that χ has relatively slow time dependence.

   (a) $\langle 4 \rangle$ Find the equation $(*)$ satisfied by $\chi(r,t)$ in the limit $c \to \infty$ and relate it to other familiar equations.

   (b) $\langle 4 \rangle$ Find the plane wave solutions of $(*)$ and use them to give approximate plane wave solutions of the Klein-Gordon equation. Mention the energy of these plane waves.

   (c) $\langle 2 \rangle$ What does the equation $(*)$ describe, in the context of the Klein-Gordon equation?

2. $\langle 2 \rangle$ Write the Klein-Gordon equation for a wave function $\psi(\vec{r},t)$ coupled to an electromagnetic field given by the potentials $\phi, \vec{A}$ in a manifestly Lorentz invariant manner.

3. $\langle 8 \rangle$ Recall that the four hermitian and traceless Dirac matrices α_i, β must each square to the identity and anti-commute in pairs. We look for a representation in terms of 2×2 matrices.

   (a) $\langle 3 \rangle$ Show that the conditions involving the α_i are met if we take α_i to be the Pauli matrices σ_i.

   (b) $\langle 5 \rangle$ Look for a matrix β to complete the set of Dirac matrices and say what you find.

4. $\langle 5 \rangle$ The four hermitian and traceless Dirac matrices α_i, β must each square to the identity and anti-commute in pairs. There are many explicit representations of the Dirac matrices. Suppose α_i, β is one such representation. Consider the transformed matrices $\beta' = U\beta U^{-1}$, $\alpha_i' = U\alpha_i U^{-1}$ for a unitary transformation U.

   (a) $\langle 3 \rangle$ Find whether α_i', β' provide another representation for Dirac matrices.

   (b) $\langle 2 \rangle$ Suppose U is not unitary but invertible. Then do α_i', β' automatically provide another representation for Dirac matrices?