Quantum Mechanics 2, Autumn 2011 CMI

Problem set 9

Due by beginning of class on Monday October 24, 2011

Semi-classical approximations, Density matrix

1. Use the improved Bohr-Sommerfeld quantization condition

$$\oint p(x)\, dx = h\left(n - \frac{\nu}{4}\right), \quad n \quad \text{a large integer} \tag{1}$$

   to find a semi-classical approximation for the highly excited energy levels of a simple harmonic oscillator $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. Here ν is the number of quantum mechanically 'penetrable' barriers encountered by the particle in the course of one closed trajectory. How does it compare with the exact result? $\langle 5 \rangle$

2. Electrons accelerated across a potential difference of 2 Volts are incident against a rectangular potential barrier of height 5 eV and of width 1 nm. (Use these values for the physical constants: mass of the electron $m = 511$ KeV/c^2, Planck's constant $h = 4.135 \times 10^{-15}$ eV.s, speed of light $c = 3 \times 10^8$ m/s and the charge of an electron is 1.6×10^{-19} Coulombs.)

   (a) What is the kinetic energy of the electrons (in electron Volts) before they hit the barrier? Compare the kinetic energy to the rest energy of the electrons. Should they be treated relativistically or would a non-relativistic approximation do? $\langle 2 \rangle$

   (b) What is the speed of electrons (as a fraction of the speed of light) before they hit the barrier? $\langle 2 \rangle$

   (c) What is the de Broglie wavelength of the incident electrons in nanometers? $\langle 2 \rangle$

   (d) Estimate the transmission probability for tunneling across the barrier. $\langle 5 \rangle$

   (e) Suppose an electron current of 1 Amp is incident on the above barrier. How many electrons per second are expected to leak through the barrier? $\langle 3 \rangle$

3. Find the density matrices ρ_1 and ρ_2 for the following two mixtures of spin half particles. (1) half the particles in $\uparrow$ and half in $\downarrow$ state and (2) half in eigenstate of S_x with eigenvalue $\hbar/2$ and half in eigenstate of S_x with eigenvalue $-\hbar/2$. $\langle 3 \rangle$

4. Would it be possible to distinguish these two mixed ensembles on the basis of ensemble averages of any observable(s)? $\langle 3 \rangle$