Quantum Mechanics 2, Autumn 2011 CMI

Problem set 8

Due by beginning of class on Monday October 17, 2011

Time reversal, Semi-classical approximations

1. Consider scattering eigenstates of positive energy $E = \frac{\hbar^2 k^2}{2m}$ for the hamiltonian of a particle in one dimension

$$H = \frac{p^2}{2m} + V(x) \qquad \text{with real potential} \quad V(x) \to 0 \quad \text{as} \quad x \to \pm\infty. \tag{1}$$

   For scattering of a unit amplitude wave incident from the far left, the wavefunction must be of the form $\psi(x,t) = \psi(x) e^{-iEt/\hbar}$ where (B and C are complex constants determined by V)

$$\psi(x) = \begin{cases} e^{ikx} + Be^{-ikx} & \text{as} \quad x \to -\infty \\ Ce^{ikx} & \text{as} \quad x \to \infty. \end{cases} \tag{2}$$

   (a) Sketch the scattering setup described by this scattering eigenstate.

   (b) Find the time reversed wave function $\psi_2(x,t) = (T\psi)(x,t)$. How does its time dependence compare with that of ψ?

   (c) Is $T\psi$ a solution of the time-dependent Schrödinger equation, and if so with what energy?

   (d) Sketch the situation described by the wavefunction $T\psi$.

   (e) Give a physical interpretation for $T\psi$ in terms of a scattering experiment in the potential $V(x)$. What does it predict?

   (f) Find a linear combination $\psi_3 = \alpha\psi + \beta T\psi$ which is a scattering eigenstate of H describing a unit amplitude wave incident from the far right. Find α and β.

2. Obtain the time-dependent Hamilton-Jacobi equation of classical mechanics from the time-dependent Schrödinger equation in the appropriate semiclassical approximation.

3. Find how time reversal acts on the momentum space wave function of a particle in one dimension $\tilde{\psi}(p,t)$.