Quantum Mechanics 1, Spring 2011 CMI

Problem set 8

Due by beginning of class on Monday March 14, 2011

Time independent Schrödinger Eqn: Particle on a circle

Consider a free particle that moves in the interval $0 \leq x \leq L$. Unlike the square well problem, here we will impose *periodic boundary conditions*: all states must satisfy $\psi(0) = \psi(L)$ and $\psi'(0) = \psi'(L)$[1].

1. Does $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ have any eigenfunctions satisfying periodic boundary conditions? If so find the orthonormal eigenfunctions and corresponding eigenvalues.

2. Do you think one could make measurements of $\hat{p}$ with arbitrary precision (in principle)? If such a measurement is made, what are the possible values of momentum that one might obtain? What is the state of the particle after one such measurement?

3. For the above particle with hamiltonian $\hat{H} = \frac{p^2}{2m}$, find the normalized energy eigenfunctions and eigenvalues. Work with exponentials rather than trigonometric functions here.

4. Can $\hat{H}$ and $\hat{p}$ be simultaneously measured with arbitrary accuracy? Why?

5. What is the ground state wave function and energy? Plot the absolute square of the ground state wave function in position space. Where along the circle (or the interval $[0, L]$) is the particle most likely to be found in the ground state?

6. How many linearly independent eigenfunctions are there at each energy level?

7. The potential here is zero, which is real. So find real energy eigenfunctions (in the position basis) at each energy level.

8. The potential here is zero, which is an even function of x about the point $x = L/2$. Find the energy eigenfunctions of definite parity (even or odd about $x = L/2$) at each energy level. Is there both an even and an odd state at each energy level?

[1] A physical realization is a particle moving on a circle: x is the coordinate along the circumference (or the polar angle ϕ), so the coordinates $x = 0$ and $x = L$ represent the same physical point (the angles $\phi = 0, 2\pi$). In this physical realization, the momentum we refer to is really the component of angular momentum L_ϕ where ϕ is the polar angle in the plane of the circle.