Quantum Mechanics 1, Spring 2011 CMI

Problem set 12

Due by beginning of class on Monday April 11, 2011

One dimensional scattering

1. Consider an attractive finite square well of with $2a$ and depth V_0 centered at $x = 0$, as treated in the lecture. Recall that a dimensionless measure of the strength of this potential is $r = a\sqrt{2mV_0}/\hbar$. In what limiting case does this potential reduce to the attractive delta function potential $-g\delta(x)$? How must r, V_0, a behave in terms of g?

2. Recall the transmission coefficient for scattering at energy $E > 0$ against an attractive finite square well of width $2a$ and depth V

$$\frac{1}{T} = 1 + \frac{1}{4}\frac{V^2}{E(E+V)}\sin^2(2la), \quad \text{where } l^2 = 2m(E+V)/\hbar^2 \tag{1}$$

The first (low energy) peak in the transmission of electrons through a gas of atoms is observed at about $E = 1$ eV. We model the potential felt by the electrons by the above finite square well with a width given by about one-tenth of a nanometer. Based on the location of the transmission peak, infer the depth of the potential due to the atomic nuclei. Hint: Electron mass = 511 keV/c^2.

3. Give the appropriate formulae for reflection and transmission coefficients R, T for scattering against a one dimensional potential which tends to zero at $-\infty$ and $V_0 > 0$ at $+\infty$. Assume that an incoming wave of amplitude A and energy $E > V_0$ is incident from the left, and is reflected with asymptotic amplitude B and transmitted with asymptotic amplitude C.

4. Find the reflection and transmission coefficients for scattering from the left at energy $E \geq V_0$ against a step barrier potential of height $V_0 > 0$ located at $x = 0$:

$$V(x) = 0 \text{ if } x < 0, \quad V(x) = V_0 \text{ if } x \geq 0. \tag{2}$$

What happens to the transmission coefficient when $E < V_0$, and why?