

Quantum Mechanics 1, Spring 2011 CMI

Problem set 10

Due by beginning of class on Monday March 28, 2011

Harmonic oscillator

1. Suppose ψ is an eigenfunction of the SHO hamiltonian with energy eigenvalue E . Show that $a\psi$ is also an eigenstate of H . What is the energy eigenvalue of $a\psi$?
2. Find the average kinetic energy $\frac{1}{2m}\langle n|p^2|n\rangle$ in the n^{th} energy level $|\psi_n\rangle = |n\rangle$ of the simple harmonic oscillator. Hint: Write p in terms of a and a^\dagger .
3. Find the average potential energy $\langle \frac{1}{2}m\omega^2x^2\rangle$ in the n^{th} energy level of the SHO.
4. Compare the results of the last two questions.
5. We model the vibrational excitations of the carbon monoxide molecule as simple harmonic oscillations of a pair of atoms (C,O) about their center of mass. Given that light of wave number $k = 2170 \text{ cm}^{-1}$ is emitted during a transition between neighboring vibrational levels, find the angular frequency of the corresponding harmonic oscillator.
6. Estimate the magnitude of (classical) acceleration experienced by one of the atoms when it has been displaced by 1nm from its equilibrium location. How does this acceleration compare with the acceleration due to gravity? Can the latter be ignored?
7. Show that the commutator satisfies the Leibnitz rule $[A, BC] = [A, B]C + B[A, C]$.