Particle Physics, Autumn 2014 CMI

Problem set 7

Due at the beginning of lecture on Tuesday Jan 20, 2015

Yukawa potential and isospin

1. ⟨**5**⟩ Evaluate the Fourier transform of a 1d Lorentzian using complex contour integration

$$V(x) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2 + m^2} \frac{dk}{2\pi} \tag{1}$$

First show that V is an even function of x so it suffices to take $x > 0$. Then use Cauchy's residue theorem to show that $V(x) = \frac{e^{-mx}}{2m}$.

2. ⟨**11**⟩ Consider the 3D Fourier transform ($k^2 = \mathbf{k} \cdot \mathbf{k}$)

$$V(\mathbf{r}) = \int_{\mathbb{R}^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(k^2 + m^2)} \frac{d^3k}{(2\pi)^3}. \tag{2}$$

   (a) ⟨**3**⟩ Argue (no calculation needed) that $V(\mathbf{r})$ is independent of the direction of $\mathbf{r}$. Hint: What happens if we rotate $\mathbf{k}$ and $\mathbf{r}$ in the same way?

   (b) ⟨**4**⟩ Perform the angular integrals and reduce $V(r)$ to a 1d integral. Show that

$$V(r) = \int_0^{\infty} \frac{\left(e^{ikr} - e^{-ikr}\right)}{ikr} \frac{1}{(k^2 + m^2)} \frac{k^2\, dk}{(2\pi)^2} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \frac{e^{ikr}}{ikr} \frac{k^2\, dk}{(k^2 + m^2)}. \tag{3}$$

   (c) ⟨**4**⟩ Evaluate the resulting 1D FT by contour integration and show that $V(r) = \frac{e^{-mr}}{4\pi r}$ is the Yukawa potential.

3. ⟨**6**⟩ 'Making' isospin 1 pions from pairs of isospin half particles. Suppose the two-component vector N transforms as an isospin doublet, i.e., $\delta N = -\frac{1}{2} i \vec{\theta} \cdot \vec{\tau} N$ where $\vec{\theta}$ are three small real numbers (encoding the axis and small angle of rotation in isospin space). Here $\tau_{1,2,3}$ are the Pauli matrices. Show that $\vec{\pi} = \frac{1}{2} N^{\dagger} \vec{\tau} N$ transforms as an $I = 1$ triplet (i.e., in the adjoint representation of SU(2): $\delta\vec{\pi} = \vec{\theta} \times \vec{\pi}$).