Particle Physics, Autumn 2014 CMI

Problem set 4

Due by beginning of lecture on Thursday Nov 6, 2014

Color, angular momentum, parity, RG, Feynman diagrams

1. $\langle \mathbf{4} \rangle$ About a third of all decays of the neutral kaon K_S^0 (which is a linear combination of K^0 and $\bar{K}^0$) is to two neutral pions $K_s^0 \to \pi^0 + \pi^0$. We assume that pions are spinless bosons and that angular momentum is conserved in this decay. Argue that the spin of the kaon cannot be an odd integer ($1, 3, 5$ etc.). Hint: Work in the rest frame of the kaon. What is the only source of angular momentum in the final state?

2. $\langle \mathbf{10} \rangle$ Consider the evolution of a coupling constant $\alpha(\mu)$ under renormalization group flow $\frac{d\alpha(\mu)}{d \log \mu} = \beta(\alpha)$ for the polynomial beta function $\beta(\alpha) = \beta_0 \alpha^2 + \beta_1 \alpha^3$ with $\beta_0 < 0$ and $\beta_1 > 0$. μ is the sliding energy scale.

   (a) $\langle \mathbf{2} \rangle$ What are the two values of α (α_0, α_*) that are invariant under the RG evolution (i.e. fixed points of the flow)?

   (b) $\langle \mathbf{2} \rangle$ Plot β as a function of α and indicate the fixed points of the RG flow on this graph.

   (c) $\langle \mathbf{3} \rangle$ Explain qualitatively what happens to $\alpha(\mu)$ as (1) $\mu \to \infty$ goes to high energies and (2) when μ is decreased from a very high energy.

   (d) $\langle \mathbf{1} \rangle$ Indicate the flow from UV (large μ) to IR (small μ) with arrows on the α axis for α between α_* and α_0.

   (e) $\langle \mathbf{2} \rangle$ Under what condition on β_0 and β_1 would weak coupling perturbation theory be expected to be reliable in this entire range of values of α? Why?

3. $\langle \mathbf{4} \rangle$ Recall the spherical harmonics $Y_{11} = N_1 e^{i\phi} \sin\theta, Y_{10} = N_0 \cos\theta, Y_{1-1} = N_{-1} e^{-i\phi} \sin\theta$ upto normalization constants N_1, N_0, N_{-1}. Check explicitly that they transform as expected under parity $\mathbf{r} \to -\mathbf{r}$.

4. $\langle \mathbf{4} \rangle$ Propagators (edges) in Feynman diagrams represent free propagation of particles as implied by the non-interacting part of the Lagrangian (quadratic terms including mass terms like $\partial_\mu \phi \partial^\mu \phi$, $m^2 \phi^2$). Emission and absorption of virtual particles by a propagating particle can lead to renormalization of its mass. Draw leading order (1-loop) diagrams for the renormalization of the Z^0 and W^- masses due to virtual quarks and leptons (two representative diagrams for each will do).

5. $\langle \mathbf{3} \rangle$ Say why a red up quark, anti-red anti-up quark state $u_{\text{red}} \bar{u}^{\text{red}}$ is not a color singlet. How would you modify this state to get one that is invariant under SU(3) color transformations?