Particle Physics, Autumn 2014 CMI
Problem set 14
Due at the beginning of lecture on Friday March 20, 2015
Klein-Gordon and Dirac fields

1. ⟨**6**⟩ The gauge-invariant generalization of the electromagnetic current of a complex scalar field coupled to an external U(1) gauge potential A^μ is

$$j^\mu = -ie\left(\phi^* D^\mu\phi - (D^\mu\phi)^*\phi\right) = -ie(\phi^*\partial^\mu\phi - (\partial^\mu\phi)^*\phi) - 2e^2A^\mu|\phi|^2 \qquad (1)$$

Show that it is conserved $\partial_\mu j^\mu = 0$, using the equation of motion $\left(\partial^2 - 2ieA\cdot\partial - ie(\partial\cdot A) - e^2A^2\right)\phi + \frac{\partial V}{\partial\phi^*} = 0$ and its c.c. that arise from the Lagrangian $(D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi)$ for a self-interacting scalar coupled to a U(1) gauge potential.

2. ⟨**2**⟩ Find $\det(\sigma^\mu p_\mu)$, where $\sigma\cdot p$ is the matrix appearing in the Pauli-Weyl equation.

3. ⟨**3**⟩ Compute $(\sigma\cdot p)^2\psi = (\sigma^\mu p_\mu)^2\psi$ where ψ satisfies the Pauli equation $p_0\psi = (\vec{\sigma}\cdot\mathbf{p})\psi$ and use it to show that each component of ψ satisfies the Klein-Gordon equation.

4. ⟨**3**⟩ Suppose ψ_a $a = 1,2,3,4$ are the 4 components of a Dirac spinor. Find $\pi^\mu_a = \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi_a}$ for the Dirac Lagrangian $\mathcal{L}_D = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi$.

5. ⟨**4**⟩ Find the conserved Noether current for the infinitesimal version of the $U(1)$ 'vector' symmetry $\psi\to e^{ie\theta}\psi, \bar\psi\to e^{-ie\theta}\bar\psi$ of the Dirac Lagrangian.

6. ⟨**5**⟩ Show that $\mathcal{L}_D = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi$ may be written as $\bar\psi_L(i\gamma^\mu\partial_\mu)\psi_L + \bar\psi_R(i\gamma^\mu\partial_\mu)\psi_R - m\bar\psi_L\psi_R - m\bar\psi_R\psi_L$. Here $\psi_{L,R} = P_{L,R}\psi = \frac{1}{2}(I\mp\gamma_5)\psi$ are the left and right chiral projections. ($P_LP_R = P_RP_L = 0$).