**Nonlinear Dynamics, Spring 2020 CMI**

Problem set 7

Due by 12 noon on Monday Mar 23, 2020

Stable and unstable manifolds, Competitive Lotka-Volterra model

1. $\langle \mathbf{3} \rangle$ What are the stable and unstable manifolds $W_s, W_u$ of a center? Why?

2. $\langle \mathbf{5} \rangle$ Suppose $(x_*, y_*)$ is part of a line (1-parameter family of fixed points). Give an example of a vector field (with formula and phase portrait) for which the stable and unstable manifolds $W_s, W_u$ of a suitable fixed point $(x_*, y_*)$ are both non-empty. What can you say about $W_s$ and $W_u$ of $(x_*, y_*)$ if the vector field is linear?

3. $\langle \mathbf{28} \rangle$ Consider the competitive Lotka-Volterra model where $r, s$ are the populations of rabbits and sheep in suitable units evolving according to

$$\dot{r} = r(3 - r - 2s) = 3r(1 - r/3) - 2rs \quad \text{and} \quad \dot{s} = s(2 - s - r) = 2s(1 - s/2) - sr. \quad (1)$$

   (a) $\langle \mathbf{3} \rangle$ Find the four fixed points in the first quadrant $r, s \geq 0$.

   (b) $\langle \mathbf{8} \rangle$ By linearizing the vector field around these fixed points, find the four Jacobian matrices. What does the linear theory predict about the nature of these fixed points. Are they hyperbolic? Can we trust these predictions?

   (c) $\langle \mathbf{5} \rangle$ Find the eigenvalues and eigenvectors of the linearization around the non-trivial fixed point.

   (d) $\langle \mathbf{8} \rangle$ Use the above results to sketch a phase portrait. Indicate all heteroclicnic and homoclinic trajectories.

   (e) $\langle \mathbf{4} \rangle$ For generic initial values of $r$ and $s$ what are the asymptotic populations? For which initial conditions can sheep and rabbits coexist as $t \to \infty$ (give the qualitative characterization of the ICs, not a formula)?