Nonlinear Dynamics, Spring 2020 CMI

Problem set 2

Due at the beginning of lecture on Monday Jan 27, 2020

Flows in 1d: fixed points and stability

1. $\langle \mathbf{6} \rangle$ Here we consider linear stability analysis for the fixed points of the overdamped driven pendulum $\dot{\theta} = \omega - a \sin\theta$ when $a > \omega > 0$. How many qualitatively different fixed points are there and where are they located? Find the growth/decay time scales by linearizing the vector field in the neighbourhood of the fixed points (only the qualitatively distinct ones).

2. $\langle \mathbf{6} \rangle$ Give examples of fixed points x_* of vector fields $v(x)$ on the line with $v'(x_*) = 0$ (so that linear stability analysis is inconclusive) but for which x_* is (a) stable (b) unstable and (c) half-stable. Illustrate with suitable phase portraits.

3. $\langle \mathbf{4} \rangle$ If x_* is a fixed point of $\dot{x} = v(x)$ with $v'(x_*) \neq 0$ we have seen that the trajectory approaches/emerges from x_* exponentially in time. The rate of approach to a fixed point can be different if $v'(x_*) = 0$. Take one of the examples from the previous answer, solve the ODE for an initial condition in the neighbourhood of x_* (or a general initial condition) and comment on the rate of approach to (or emergence from) equilibrium.