Nonlinear Dynamics, Spring 2020 CMI

Problem set 11

Due by 12 noon on Tuesday April 7, 2020

Index of a vector field

1. ⟨**7**⟩ Suppose C is a simple closed curve (no self intersections) that encloses n isolated fixed points $\mathbf{r}_{1*}, \mathbf{r}_{2*}, \ldots, \mathbf{r}_{n*}$ with indices $I_{\mathbf{r}_{1*}}, \ldots, I_{\mathbf{r}_{n*}}$. Show that the index of C is the sum of the indices of the enclosed fixed points.

$$I_C = I_{\mathbf{r}_{1*}} + I_{\mathbf{r}_{2*}} + \cdots + I_{\mathbf{r}_{n*}}. \tag{1}$$

   Hint: Draw a figure showing the fixed points and contour and try to deform the contour and argue what happens.

2. ⟨**7**⟩ Find the index of the fixed point at the origin of the dipole field $\dot{x} = x^2 - y^2$, $\dot{y} = 2xy$. Sketch the vector field and choose a suitable contour C. Hint: You may answer this question purely by pictorial methods without evaluating any integrals.