Nonlinear Dynamics, Spring 2020 CMI

Problem set 10

Due by 5pm on Friday April 3, 2020

Index of a vector field

1. $\langle \mathbf{4} \rangle$ Calculate the index of a vector field (around a unit circle centered at the origin and oriented counterclockwise) with constant cartesian components.

2. $\langle \mathbf{3} \rangle$ Pictorially argue what the index of a vector field is whose cartesian components fluctuate a little bit. Draw a figure of the proposed vector field and curve C.

3. $\langle \mathbf{6} \rangle$ Consider the linear system $\dot{x} = ax$ and $\dot{y} = ay$ for $a \neq 0$. What sort of fixed point does the corresponding vector field $\mathbf{v}$ have at the origin? Take C to be the unit circle centered at the origin and traversed counter-clockwise. Find the index $I_C(\mathbf{v})$. Use a suitable parametrization of C to calculate the index. How does $I_C(\mathbf{v})$ depend on the sign of a, briefly comment.

4. $\langle \mathbf{6} \rangle$ Consider the linear oscillator $m\ddot{x} = -kx$ with $m = k = 1$. Choose C to be a unit circle traversed counter-clockwise. Find the index I_C of the corresponding vector field. Illustrate with a figure and also obtain the answer pictorially.