

Nonlinear Dynamics, Spring 2019 CMI

Problem set 2

Due at the beginning of lecture on Tuesday Jan 29, 2019

Flows in 1d

1. **(6)** Give examples of fixed points x_* of vector fields $v(x)$ on the line with $v'(x_*) = 0$ (so that linear stability analysis is inconclusive) but for which x_* is (a) stable (b) unstable and (c) half-stable. Illustrate with suitable phase portraits.
2. **(4)** If x_* is a fixed point of $\dot{x} = v(x)$ with $v'(x_*) \neq 0$ we have seen that the trajectory approaches/emerges from x_* exponentially in time. The rate of approach to a fixed point can be different if $v'(x_*) = 0$. Take one of the examples from the previous answer, solve the ODE for an initial condition in the neighbourhood of x_* (or a general initial condition) and comment on the rate of approach/emergence to/from equilibrium.
3. **(6)** Solve the following inhomogeneous linear ODE for $x(t)$:

$$\dot{x} = \lambda x + y_0 e^{\lambda t} \quad \text{with} \quad x(0) = x_0. \quad (1)$$

Here y_0 is a constant and $\lambda \neq 0$ is a real constant. Give the intermediate steps in obtaining your solution. Check that it satisfies the IVP.