Mathematical Physics 1: Linear Algebra, CMI

Problem set 7

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Due at the beginning of class on Friday, August 28.

Projections

1. The projection onto a subspace $U_d \in V_n$ should depend only on the subspace U and not on a basis we pick for it. In lecture we showed this for projection onto a 1-dimensional subspace. You will show this here in general. Suppose the columns $\vec{a}_1, \vec{a}_2, \cdots, \vec{a}_d$ of A are a basis for U and the columns of B are also a basis for U. Then they are related by a change of basis C. In particular we can expand each of the b-basis vectors in the a-basis as in $b_1 = c_{11}a_1 + c_{21}a_2 + \cdots c_{d1}a_n$. Proceed in this manner and write the matrix C (explicitly) such that $B = AC$.

2. What are the dimensions of A, B and C?

3. Is C invertible and why?

4. Show that $P_A = P_B$ using the formula for the projection to a subspace spanned by the columns of a matrix. (Hint: Use the fact that $A^T A$ is invertible.)

5. In lecture we showed that if $A_{n\times d}$ has linearly independent columns, then $A^T A$ is invertible. Here you will show the converse. Suppose we know that $A^T A$ is invertible i.e. has trivial kernel. Then argue why A must have linearly independent columns. (Hint: The proof is in a similar spirit to the one given in lecture and should not be more than a few lines.)

6. Suppose $\vec{q}_i$ form an orthonormal basis for a vector space and v is a vector in this space. Then what is the orthogonal projection of v onto the basis vector $\vec{q}_j$?