Mathematical Physics 1: Linear Algebra, CMI

Problem set 5

Instructor: Govind S. Krishnaswami

Due at the beginning of class on Friday, August 21.

Transpose, Inverse, Linear transformation

1. Find the inverse of the matrix Q using Gauss-Jordan elimination, and say when it exists (θ is a real number. Check your answer against the general formula for inverse of a 2×2 matrix obtained in lecture.)

$$Q = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1}$$

2. Between which two vector spaces is Q a linear transformation?

3. Find the transpose of Q, and comment on its relation to Q^{-1}

4. Is Q an isomorphism?

5. Plot the action of Q on the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the plane for $\theta = \pi/4$.

6. Give a suitable name/description for Q that describes its action on vectors.

7. Consider the reflection R of any vector in $\mathbf{R}^2$ about the x-axis. Write in components what R does to a general vector.

8. Is R a linear transformation? Why?

9. If it is a linear transformation, find the matrix representation of the reflection R in the standard cartesian basis for $\mathbf{R}^2$.

10. The matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$ is a toy version of the annihilation operator in quantum mechanics. Find
    (a) its rank,
    (b) its pivots and determinant
    (c) all vectors it annihilates
    (d) a 3-component column vector b for which $Ax = b$ has no solution.