Mathematical Physics 1: Linear Algebra, CMI

Problem set 3

Instructor: Govind S. Krishnaswami

Due at the beginning of class on Friday August 14.

Gaussian Elimination

1. Write the following linear equations in matrix form $Ax = b$ and use elementary row operations to reduce to row echelon form (upper triangular form).

$$2x + 3z = -1, \quad x - y + 5z = 1, \quad 4y - 2z = 6 \tag{1}$$

2. What are the pivots?
3. What is the rank of A?
4. Is A invertible?
5. Continue with elimination upwards to bring A to diagonal form. Find all solutions x, y, z. Check that they are correct by plugging into the original system of equations.
6. What are the elementary matrices E_{21}, E_{31}, E_{32} needed for reduction to echelon form?
7. What are the inverses of E_{21}, E_{31}, E_{32}?
8. Factorize $A = LU$ as the product of a lower and upper triangular matrix.
9. What is the determinant of A? (Look up how to calculate the determinant of a 3×3 matrix if you need to.)
10. What is the product of the pivots of A?
11. What is the permutation matrix C that cyclically permutes the rows of a 3×3 matrix? i.e., $r_1 \to r_2 \to r_3 \to r_1$?
12. Write the above cyclic permutation matrix C as a product of row exchange matrices. (i.e., think how you can realize a cyclic permutation via successive row exchanges.)