

Mathematical Physics 1: Linear Algebra, CMI

Problem set 12

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Due at the beginning of class on Fri, Sep 18.

Principal Axis Transformation

1. Consider the quadratic curve E in the $x - y$ plane defined by the equation $2x^2 + 3xy - yx + 2y^2 = 1$. Write this equation as a matrix equation and identify the real symmetric matrix A whose quadratic form is involved.
2. Plot the curve E roughly on the $x - y$ plane. (Find a few points on E and join the dots, the figure must show the major and minor axes roughly)
3. Do the $x - y$ coordinate axes point along the principal axes of E ? Why or why not?
4. What is the condition for the position vector of a point P to point in the same direction as the normal?
5. Find the principal axes of E by interpreting it as an eigenvalue problem.
6. Find the lengths of the semi-major and semi-minor axes.
7. Indicate the principal axes and their lengths in a figure.
8. Find the particular principal axis transformation Q for the above quadratic curve satisfying $\det Q = +1$. What sort of transformation is Q , describe its action on the coordinate axes? (Hint: This and the next question involve choices of order!)
9. Find a different principal axis transformation Q' with $\det Q' = -1$. Describe the action of Q' on the coordinate axes.
10. Explain the need for reflections in the passage to principal axes and in the choice of eigenvalue matrix Λ in the above example.
11. Is A a positive definite matrix? Why?
12. Find e^A for the above matrix A using the principal axis transformation.