Mathematical Physics 1: Linear Algebra, CMI

Problem set 10

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Due at the beginning of class on Tue, Sep 8.

Determinants, Eigenvalues and Eigenvectors

1. Consider the change of coordinates from spherical polar to cartesian coordinates in three dimensional Euclidean space $(r, \theta, \phi) \mapsto (x, y, z)$ where $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$
$$z = r\cos\theta, \quad x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi. \tag{1}$$
Draw a diagram indicating $(x, y, z), (r, \theta, \phi)$ for a point in the interior of the first quadrant.

2. Find the Jacobian matrix J for the transformation of coordinates.

3. The change in volume element is $dxdydz = dr\ d\theta\ d\phi\ \det J$. Find $\det J$ using cofactors.

4. Suppose A is upper triangular and invertible. Use the cofactor formula for A^{-1} to explain what sort of matrix A^{-1} is.

5. Consider the matrix $Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ representing a linear transformation $Q : R^2 \to R^2$. Describe what it does to vectors.

6. Does Q have any real eigenvectors? Why? (Answer this question with out explicit calculations)

7. Find the eigenvalues of Q.

8. Now regard Q as a linear transformation $Q : C^2 \to C^2$. Find eigenvectors (with norm $= 1$) corresponding to the above eigenvalues.