Continuum Mechanics, Spring 2018 CMI

Problem set 7

Due at the beginning of lecture on Tuesday Mar 27, 2018

Tensor of Elasticity and Elastic Potential Energy

1. $\langle \mathbf{7} \rangle$ Count the number of components of the tensor of elasticity Y_{ijkl} for a material in three dimensions with the following properties.

   (a) $\langle \mathbf{1} \rangle$ Having all indices the same.

   (b) $\langle \mathbf{2} \rangle$ Having precisely one lone index. (e.g. Y_{xyyy})

   (c) $\langle \mathbf{2} \rangle$ Having precisely two lone indices. (e.g. Y_{xyzz} or Y_{yxzx})

   (d) $\langle \mathbf{2} \rangle$ Having 2 distinct indices each repeated twice.

2. $\langle \mathbf{6} \rangle$ Recall that the tensor of elasticity Y for an isotropic material $Y_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj})$ may be regarded as a symmetric operator on the 6d space of symmetric 3×3 strain tensors.

   (a) $\langle \mathbf{3} \rangle$ Show that the Kronecker delta is an eigenvector of Y. Find the corresponding eigenvlaue.

   (b) $\langle \mathbf{3} \rangle$ Find the trace of Y.

3. $\langle \mathbf{8} \rangle$ Express the elastic potential energy U of an isotropic material occupying a volume V in terms of the expansion Θ, shear tensor Σ and the bulk and shear moduli K and μ.