Continuum Mechanics, Spring 2018 CMI

Problem set 5

Due at the beginning of lecture on Monday Mar 12, 2018

Elasticity: Strain tensor

1. $\langle\mathbf{13}\rangle$ Consider the second-rank tensor field $S = \nabla\xi$ for a displacement field ξ with S_{ij} in Cartesian coordinates given by $\epsilon \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ where ϵ is a small dimensionless parameter.

   (a) $\langle\mathbf{3}\rangle$ Find the corresponding rotation/vorticity tensor ω_{ij}, strain tensor e_{ij}, shear tensor Σ_{ij} and expansion Θ.

   (b) $\langle\mathbf{6}\rangle$ Find a three dimensional displacement field ξ corresponding to the above S. What is the curl of ξ?

   (c) $\langle\mathbf{4}\rangle$ Plot the vector field ξ in the x-y plane and mention the type of elastic deformation it corresponds to. You must indicate the direction of the vector field by arrows.

2. $\langle\mathbf{15}\rangle$ Consider a homogeneous isotropic material with Young's modulus E and Poisson's ratio ν. It is subject to a horizontal (along x) normal tensile stress g everywhere.

   (a) $\langle\mathbf{6}\rangle$ Find an expression for the displacement field in the material in Cartesian coordinates. Hint: Consider a bar of rectangular cross-section with faces parallel to the coordinate planes and choose a suitable origin for coordinates.

   (b) $\langle\mathbf{5}\rangle$ Find the corresponding tensor field $S = \nabla\xi$ and decompose it into a rotation ω_{ij} and strain tensor e_{ij}.

   (c) $\langle\mathbf{4}\rangle$ Find the corresponding shear tensor Σ and expansion Θ.

3. $\langle\mathbf{7}\rangle$ Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are three-component row vectors and $\delta\mathbf{u}, \delta\mathbf{v}, \delta\mathbf{w}$ are small 3-component row vectors. Consider the 3×3 matrix $M = \begin{pmatrix} \mathbf{u} + \delta\mathbf{u} \\ \mathbf{v} + \delta\mathbf{v} \\ \mathbf{w} + \delta\mathbf{w} \end{pmatrix}$. Find a simple expression for $\det M$ as a sum of determinants by dropping terms that are quadratic or higher order in small quantities. Justify your result.