Continuum Mechanics, Spring 2018 CMI
Problem set 4
Due at the beginning of lecture on Monday Mar 5, 2018
Elastostatics

1. ⟨**8**⟩ We obtained the stress tensors associated to hydrostatic pressure and tensile force on a bar. Here we consider the stress tensor for a material where any elementary cube is subject to pure shear forces of size g per unit area. As in the example considered in lecture, the stress g due to an external agent acts tangent to the top, bottom, right and left faces in the $\hat{x}, -\hat{x}, \hat{y}$ and $-\hat{y}$ directions. Draw a diagram indicating the cube and the forces. By considering the forces acting across these faces, deduce the stress tensor T_{ij} in the standard Cartesian basis referred to above.

2. ⟨**10**⟩ Diagonalize the pure shear stress tensor obtained above by finding an orthogonal transformation S such that $S^{-1}TS = \tilde{T}$ is diagonal. Find S and $\tilde{T}$. Physically interpret the new stress tensor $\tilde{T}$ by specifying the sort of forces it represents acting across appropriate surfaces.

3. ⟨**8**⟩ Decompose a 2nd rank real tensor with components t_{ij} $(1 \leq i, j \leq N \geq 2)$ as a sum of traceless symmetric, anti-symmetric and scalar parts: $t_{ij} = s_{ij} + a_{ij} + \theta_{ij}$. Give formulae for s, a and θ. How many independent components do t, s, a and θ possess as a function of N?

4. ⟨**5**⟩ Estimate the pressure on the surface of the Earth. You may suppose that it is due to a column of air of height about 10 km of roughly constant density 1 kg/m^3. You may suppose that the acceleration due to gravity g does not change much over this height, from its value on the Earth's surface. Give the answer in Pascals as well as dynes per square centimeter.