Classical Mechanics 2, Spring 2016 CMI
Problem set 5
Due by the beginning of lecture on Monday Feb 8, 2016
Poisson Brackets

1. $\langle \mathbf{4} \rangle$ Find the *unequal* time p.b. $\{q(0), q(t)\}$ for a free particle of mass m moving on a line. Hint: Use the solution of the equation of motion.

2. $\langle \mathbf{14} \rangle$ Angular momentum Poisson brackets from $\{r_i, p_j\} = \delta_{ij}$. Recall from class discussion that the Cartesian components of angular momentum may be expressed as $L_i = \epsilon_{ijk} r_j p_k$. We place all indices down-stairs in this problem, and sum repeated indices.

   (a) $\langle \mathbf{7} \rangle$ Use the properties of the Poisson bracket and the identity

   $$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}. \tag{1}$$

   to show that

   $$\{L_i, L_j\} = r_i p_j - r_j p_i. \tag{2}$$

   For uniformity of notation, begin by taking $L_i = \epsilon_{ikl} r_k p_l$ and $L_j = \epsilon_{jmn} r_m p_n$

   (b) $\langle \mathbf{2} \rangle$ Use the above formula for L_i to show that $\epsilon_{ijk} L_k = r_i p_j - r_j p_i$. Thus we conclude that $\{L_i, L_j\} = \epsilon_{ijk} L_k$.

   (c) $\langle \mathbf{3} \rangle$ Use the above results to show that

   $$\{\{L_i, L_j\}, L_k\} = \delta_{ik} L_j - \delta_{jk} L_i. \tag{3}$$

   (d) $\langle \mathbf{2} \rangle$ Show that the components of angular momentum satisfy the Jacobi identity

   $$\{\{L_i, L_j\}, L_k\} + \{\{L_j, L_k\}, L_i\} + \{\{L_k, L_i\}, L_j\} = 0. \tag{4}$$