Classical Mechanics 2, Spring 2016 CMI

Problem set 4

Due by the beginning of lecture on Monday Feb 1, 2016

Hamiltonian from Lagrangian, Legendre transform

1. $\langle \mathbf{6} \rangle$ The Lagrangian of a charged particle in a magnetic field (given by the vector potential $\vec{A}(\vec{q})$) is $L = \frac{1}{2} m \dot{\vec{q}}^2 + (e/c) \vec{A} \cdot \dot{\vec{q}}$.
   (a) $\langle \mathbf{1} \rangle$ Find the momentum p_i conjugate to q_i.
   (b) $\langle \mathbf{3} \rangle$ Find the Hamiltonian H as a function of $\vec{q}$ and $\vec{p}$ by computing the Legendre transform.
   (c) $\langle \mathbf{2} \rangle$ Express H as the dot-product of a vector with itself.

2. $\langle \mathbf{5} \rangle$ The first law of thermodynamics says that the increase in internal energy of a gas is equal to the heat supplied to the gas minus the work done by the gas. For infinitesimal reversible changes, $dU = TdS - PdV$. Here dU is the increase in internal energy, P the pressure, dV the increase in volume dS the increase in entropy and T the absolute temperature.
   (a) $\langle \mathbf{1} \rangle$ What are the independent variables that U depends on?
   (b) $\langle \mathbf{1} \rangle$ Write formulae to determine the temperature and pressure from the internal energy
   (c) $\langle \mathbf{1} \rangle$ Helmholtz free energy may be introduced via the formula $F = U - TS$. Find the independent variables that F depends on.
   (d) $\langle \mathbf{1} \rangle$ Express the pressure and entropy in terms of the Helmholtz free energy.
   (e) $\langle \mathbf{1} \rangle$ Write a formula for Helmholtz free energy as a Legendre transform of the internal energy. Indicate which variable to extremize in and give the condition for an extremum.

3. $\langle \mathbf{5} \rangle$ Given a Hamiltonian $H(q,p)$, in favourable cases, one may obtain the corresponding Lagrangian $L(q,\dot{q})$ by a suitable 'inverse' Legendre transform. Give a formula to get L from H. Mention which variable to extremize in and what the condition for an extremum is. Relate the condition for an extremum to a familiar equation.