Classical Mechanics 2, Spring 2016 CMI
Problem set 1
Due by the beginning of lecture on Monday Jan 11, 2016
Conserved energy, simple harmonic oscillator

1. $\langle \mathbf{4} \rangle$ Derive a conserved energy for Newton's equation for three degrees of freedom $m\ddot{x}_i = f_i$ where $i = 1,2,3$ or $m\ddot{\mathbf{r}} = \mathbf{f}$ where the cartesian components of the force are $f_i = -\frac{\partial V}{\partial x_i}$. Proceed by finding a suitable integrating factor.

2. $\langle \mathbf{8} \rangle$ Recall that for motion of a particle of mass m on a line, the solutions $x(t)$ of Newton's equation with energy E and initial position x_0 was reduced to the integral

$$t - t_0 = \pm \int_{x_0}^{x} \frac{dy}{\sqrt{\frac{2}{m}(E - V(y))}} \qquad (1)$$

Consider a simple harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$ for which $E \geq 0$ and let $\omega = \sqrt{\frac{k}{m}}$.

(a) $\langle \mathbf{4} \rangle$ Evaluate the integral (use $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u$) and solve for the trajectories with given E, x_0. Show that you get

$$x(t) = \pm\sqrt{\frac{2E}{k}} \sin\left(\omega(t-t_0) \pm \arcsin\left(\sqrt{\frac{k}{2E}}\, x_0\right)\right). \qquad (2)$$

The upper signs correspond to one solution and the lower signs to another solution.

(b) $\langle \mathbf{2} \rangle$ Specialize to the case where the particle starts from the equilibrium position at $t_0 = 0$ and simplify the formula for $x(t)$. Also find the momentum $p(t)$.

(c) $\langle \mathbf{2} \rangle$ Show that $x(t)$ satisfies the 'initial conditions' $x(0) = 0$ and $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = E$.