Classical Mechanics 2, Spring 2014 CMI
Problem set 9
Due by the beginning of lecture on Monday Mar 10, 2014
Anharmonic oscillator and ϵ symbol.

1. $\langle$**18**$\rangle$ Consider a particle of mass m moving subject to the double well potential $V(x) = g(x^2 - a^2)^2$ with $g, a > 0$.

   (a) $\langle$**3**$\rangle$ Suppose we consider a non-static solution with energy $E = ga^4$, where the trajectory lies in the left well. Find the left turning point x_m of such a trajectory and indicate E, x_m in a graph of the potential.

   (b) $\langle$**5**$\rangle$ Obtain the following expression for the time taken by the particle to go from x_m (starting at rest) to $x = 0$
   $$T = \sqrt{\frac{m}{2g}} \int_{x_m}^{0} \frac{dx}{\sqrt{2x^2a^2 - x^4}}. \quad (1)$$

   (c) $\langle$**4**$\rangle$ Identify where in the interval $x_m \leq x \leq 0$ the integrand is singular (i.e. diverges). Roughly plot the integrand as a function of x in this interval.

   (d) $\langle$**6**$\rangle$ Show that $T = \infty$ by considering the leading behavior of the integrand near its singularities. Which singularity is integrable and which is not? Do this *without evaluating the indefinite integral explicitly*. Conclusion: a particle released from rest at x_m takes infinitely long to reach $x = 0$ and cannot cross the barrier.

2. $\langle$**6**$\rangle$ It is possible to argue that the contraction of two ϵ symbols given below should be expressible as a linear combination of products of Kronecker deltas:
   $$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = a\delta_{jk}\delta_{lm} + b\delta_{jl}\delta_{km} + c\delta_{jm}\delta_{kl} \quad \forall \quad 1 \leq j,k,l,m \leq 3. \quad (2)$$

   Find the constants a, b, c using the known properties and values of ϵ and δ.