Classical Mechanics 2, Spring 2014 CMI

Problem set 6

Due by the beginning of lecture on Monday Feb 3, 2014

Hamiltonian from Lagrangian by Legendre transform

1. ⟨**4**⟩ The first law of thermodynamics says that the change in internal energy of a gas is equal to the heat supplied to the gas minus the work done by the gas. For infinitesimal reversible changes, $dU = TdS - PdV$. Here dU is the increase in internal energy, P the pressure, dV the increase in volume dS the increase in entropy and T the absolute temperature.

   (a) ⟨**1**⟩ What are the independent variables that U depends on?

   (b) ⟨**1**⟩ Helmholtz free energy may be introduced via the formula $F = U - TS$. Find the independent variables that F depends on.

   (c) ⟨**1**⟩ Express the pressure and entropy in terms of the Helmholtz free energy.

   (d) ⟨**1**⟩ Write a formula for Helmholtz free energy as a Legendre transform of the internal energy. Indicate which variable to extremize in and give the condition for an extremum.

2. ⟨**7**⟩ Find a smooth convex function $L(v)$ of a real variable v whose Legendre transform $H(p) = \mathrm{ext}_v(pv - L(v))$ is the same function as L. In other words, $H(x) = L(x)$ for any $x \in \mathbb{R}$. You may first try to guess such a function using physical knowledge. But you must also formulate the above condition as an equation and try to solve it to determine such a function. Give a physical interpretation of the resulting function $H(p)$.