Classical Mechanics 2, Spring 2014 CMI

Problem set 13

Due by the beginning of lecture on Monday Apr 7, 2014

Force free motion of a symmetric top, Euler angles

1. ⟨**12**⟩ Consider force free motion of a symmetric top with $I_1 = I_2$, as discussed in the lecture. Suppose the axis of the top makes an angle $\theta \neq 0$ with the fixed direction of $\mathbf{L}$.

   (a) ⟨**6**⟩ Find the angle α between the angular velocity vector $\mathbf{\Omega}$ and angular momentum vector $\mathbf{L}$ (α is half the opening angle of the cone swept out by $\mathbf{\Omega}$). Express α in terms of θ, the principal moments of inertia and the magnitude of angular momentum L. How does α depend on time and L?

   (b) ⟨**3**⟩ Suppose $I_1 \to I_3$ so that the symmetric top becomes a spherical top. Based on our study of the spherical top, what do you expect to happen to α? Is this expectation fulfilled by the above formula for α?

   (c) ⟨**3**⟩ It can be shown that to take the limit of a rigid rotator (starting from a symmetric top), $\cos\theta$ must tend to zero faster than I_3. Using this, find the limiting value of α for a rigid rotator. Does it agree with the value obtained in our study of a rigid rotator?

2. ⟨**5**⟩ Euler angles θ, ϕ, ψ, were defined in the lecture (see the lecture notes). Express the generalized velocities $\dot{\theta}, \dot{\phi}, \dot{\psi}$ in terms of the angular velocity components $\Omega_1, \Omega_2, \Omega_3$.

3. ⟨**6**⟩ Consider force free rotational motion of a symmetric top ($I_1 = I_2 \neq I_3$) described in terms of Euler angles. Let the co-rotating axes x, y, z by chosen along principal axes of inertia. We follow the notation adopted in the lecture and notes. Write the Lagrangian and find the momenta conjugate to the Euler angles and identify which are conserved. From the figure, identify which among p_θ, p_ϕ, p_ψ corresponds to the Z component of angular momentum L_Z, and which corresponds to the z component L_z.