Classical Mechanics 1, Autumn 2022 CMI

Problem set 6

Due by 6pm, Thu Sep 29, 2022

Dimensional analysis, Newton's laws, Galilean Relativity, conservative force, energy

1. $\langle \mathbf{4} \rangle$ Consider a system of two point particles A and B of masses m_a and m_b. A exerts a force $\boldsymbol{F}_b$ on B and B exerts a force $\boldsymbol{F}_a$ on A. Write Newton's 2nd law equations of motion for this system in an inertial frame where the position vectors of the particles are $\boldsymbol{r}_a$ and $\boldsymbol{r}_b$. Use Newton's laws to show that the total momentum of the system is independent of time.

2. $\langle \mathbf{4} \rangle$ Suppose events A and B occur at the same location $\boldsymbol{r}_0$ but at distinct times $t_A < t_B$, as observed in an inertial frame S. Do they occur at the same location in all other inertial frames that are in uniform motion relative to S? Justify your answer using formulae.

3. $\langle \mathbf{5} \rangle$ Find the dimensions ($M^\alpha L^\beta T^\gamma$) of (a) Newton's gravitational constant G, (b) Planck's constant h from the formula $E = h\nu$ where E and ν are the energy and frequency of a photon, (c) GM_e/c^2 where M_e is the mass of the Earth and c the speed of light and (d) the probability amplitude $\psi(x)$ for finding an electron within dx of x, given that the total probability $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ for finding the electron somewhere on the real line is one.

4. $\langle \mathbf{8} \rangle$ Consider the matrix $A = \begin{pmatrix} 0 & 1/m \\ -k & 0 \end{pmatrix}$. The matrix exponential e^{At} is defined as $e^{At} = \sum_{p=0}^{\infty} \frac{(At)^p}{p!}$. Here t is time. Calculate e^{At} and express the answer explicitly as a 2×2 matrix whose entries are (multiples of) trigonometric functions. Use the abbreviation $\omega = \sqrt{k/m}$ where convenient. You may use the results of Problem Set 5.

5. $\langle \mathbf{2+2} \rangle$ Suppose a particle moves along the real line (with coordinate x) and subject to a conservative force $f(x) = -V'(x)$. Though f and x are vectors, it is conventional to omit vector notation, since the motion is along a straight line. (a) $\langle \mathbf{2} \rangle$ Given f, is the potential uniquely determined? Explain. (b) $\langle \mathbf{2} \rangle$ Suppose the restoring force in a spring elongated by x is given by $f(x) = -kx$ where $k > 0$ is a fixed force constant. Find a potential $V(x)$ corresponding to this force. Plot $V(x)$ as a function of x labeling the axes.

6. $\langle \mathbf{4} \rangle$ Use an appropriate integrating factor to derive a conserved energy for Newton's equation $m\ddot{\boldsymbol{r}} = \boldsymbol{f}$ for a particle of mass m moving in three dimensions subject to a potential V. The corresponding conservative force has Cartesian components $f_i = -\frac{\partial V}{\partial x_i}$ for $i = 1, 2, 3$.