Classical Mechanics 1, Autumn 2022 CMI
Problem set 5
Due by 6pm, Monday Sep 12, 2022
Angular momentum, degrees of freedom, Newton's 2nd law

1. $\langle \mathbf{6} \rangle$ Consider a particle moving in 3d Euclidean space. Its position vector is $\boldsymbol{r}$ relative to a fixed origin and its momentum is $\boldsymbol{p} = m\dot{\boldsymbol{r}}$ where m is its mass. Its angular momentum with respect to the chosen origin is defined as $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$. The Cartesian components of $\boldsymbol{L}$ are denoted L_x, L_y, L_z. In spherical polar coordinates (defined in the lecture), show that the z component of angular momentum L_z of the particle can be expressed as $L_z = mr^2 \sin^2\theta\,\dot{\phi}$.

2. $\langle \mathbf{5} \rangle$ A point particle of mass m is observed to move counterclockwise round a circle of radius ℓ at a constant speed v. Use Newton's second law to infer the net force (magnitude and direction) that must be acting on the particle, assuming it is observed in an inertial frame.

3. $\langle \mathbf{4} \rangle$ Suppose a particle moves on a spherical surface of radius $R = 1$ fixed in an inertial frame. (a) How many degrees of freedom does it have? (b) How many real numbers must be specified to provide initial conditions to solve Newton's equation of motion for this particle? (c) Suggest possible physical quantities that could be used to specify these initial conditions.

4. $\langle \mathbf{7} \rangle$ Consider a particle of mass m moving on the real line (x-axis) subject to the force $\boldsymbol{F} = -kx\hat{x}$ for a force constant $k > 0$. (a) Write down Newton's equation of motion for this particle. (b) Express the equation as a pair of first order equations for $x(t)$ and the momentum $p(t)$ in matrix form

$$\frac{d}{dt}\begin{pmatrix} x \\ p \end{pmatrix} = A \begin{pmatrix} x \\ p \end{pmatrix}. \tag{1}$$

   Find the 2×2 matrix A. (c) Find A^2, A^3, A^4 and propose a formula for A^{2n} and A^{2n+1} for $n = 0, 1, 2, \ldots$.