Classical Mechanics 1, Autumn 2022 CMI

Problem set 4

Due by 6pm, Monday Sep 5, 2022

Spherical polars, gradient, degrees of freedom, Newton's first law

1. $\langle \mathbf{9} \rangle$ Consider the scalar field $\psi = 1/\sqrt{x^2+y^2+z^2}$ defined using Cartesian coordinates in $\mathbb{R}^3$ with the origin left out. (a) Find the gradient of ψ for x, y, z not all zero. After obtaining the answer in Cartesian coordinates, re-express ψ and the final answer for the gradient in spherical polar coordinates. (b) Say in words which way $\nabla\psi$ points and why and how its magnitude varies. Plot this vector field on a suitably chosen plane through the origin. Does the choice of plane matter? Why or why not?

2. $\langle \mathbf{5} \rangle$ Consider an idealized straight rigid wire of zero thickness and fixed length ℓ (essentially a line segment) that is free to move in the seminar hall of CMI. How many degrees of freedom does this system possess? Explain by enumerating the degrees of freedom.

3. $\langle \mathbf{4} \rangle$ Suppose a particle is known to be isolated (i.e., not subject to any force) but observer A finds that it executes accelerated motion with respect to A's frame of reference. On the basis of Newton's 1^{st} law, explain whether A's frame is inertial or not.

4. $\langle \mathbf{4} \rangle$ Suppose a particle is known to be subject to a net force but an observer finds that it does not move. (a) Based on Newton's 1^{st} law, what can be deduced about whether the observer's frame is inertial or noninertial? (b) Based Newton's 2^{nd} law, what can be said about whether the observer's frame is inertial or noninertial?