Classical Mechanics 1, Autumn 2022 CMI
Problem set 2
Due by 6pm, Wednesday Aug 17, 2022
Scalar and vector products, Circular motion

1. $\langle\mathbf{4}\rangle$ Express the Cartesian components of the cross product $\boldsymbol{a} \times \boldsymbol{b}$ in terms of those of $\boldsymbol{a}$ and $\boldsymbol{b}$.

2. $\langle\mathbf{4}\rangle$ Show that $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b})$ by working with the Cartesian components of the three vectors.

3. $\langle\mathbf{3}\rangle$ Define the Levi-Civita symbol ϵ_{ijk} for integers $1 \leq i, j, k \leq 3$ (not necessarily distinct) by the conditions (i) $\epsilon_{123} = 1$ and (ii) ϵ_{ijk} reverses sign under exchange of any pair of indices [e.g. $\epsilon_{ijk} = -\epsilon_{jik}$]. Find ϵ_{213}, ϵ_{312}, ϵ_{321}, ϵ_{112}, ϵ_{313} and ϵ_{222}.

4. $\langle\mathbf{4}\rangle$ Verify that $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \sum_{1 \leq i,j,k \leq 3} \epsilon_{ijk} a_i b_j c_k$ where a_i, b_j, c_k are the Cartesian components of three vectors in $\mathbb{R}^3$. Here $a_1 = a_x, a_2 = a_y, a_3 = a_z$ etc.

5. $\langle\mathbf{4}\rangle$ Suppose $\boldsymbol{a}(t)$ is the acceleration vector of a particle undergoing uniform circular motion at angular speed ω counterclockwise around a circle of radius ℓ with position vector $\boldsymbol{r}$ relative to the center of the circle. The circle is centered at the origin of the x-y plane. Suppose the particle is at $x = \ell, y = 0$ at $t = 0$. Find $\boldsymbol{v} = \dot{\boldsymbol{r}}, \boldsymbol{a}$ and $\dot{\boldsymbol{a}}$ (as linear combinations of $\hat{x}$ and $\hat{y}$). Show that $\dot{\boldsymbol{a}} \cdot \boldsymbol{a} = 0$. What is the significance of this result?