Classical Mechanics (PG), Autumn 2013 CMI
Problem set 4
Due at the beginning of lecture on Wednesday August 21, 2013
Lagrangian mechanics

1. $\langle \mathbf{8} \rangle$ Consider a particle whose dynamics is specified by the Lagrangian

$$L(q, \dot{q}) = \frac{1}{2} m \dot{q}^2 + b(q) \dot{q}. \quad (1)$$

Here $b(q)$ is some differentiable function of q.

(a) $\langle \mathbf{4} \rangle$ What sort of motion does the Lagrangian describe? Find any trajectories.

(b) $\langle \mathbf{4} \rangle$ Explain the nature of this particle by examining the principle of extremal action $S = \int_{t_0}^{t_1} L \, dt$ for this Lagrangian. Can you relate this action to a more familiar one?

2. $\langle \mathbf{10} \rangle$ Suppose we consider a system with generalized coordinates q^i, potential energy $V(q)$, kinetic energy $T = \frac{1}{2} g_{ij}(q) \dot{q}^i \dot{q}^j$ and total energy $E = T + V$.

(a) $\langle \mathbf{2} \rangle$ Argue that we may take $g_{ij}(q)$ to be a symmetric tensor. It is a sort of position-dependent mass matrix.

(b) $\langle \mathbf{2} \rangle$ Suppose the Lagrangian is

$$L = T - V = \frac{1}{2} g_{ij}(q) \dot{q}^i \dot{q}^j - V(q) \quad (2)$$

Find a simple formula for the momenta p_k conjugate to q^k.

(c) $\langle \mathbf{3} \rangle$ Find the hamiltonian $H = p_k \dot{q}^k - L$ and relate it to the energy E.

(d) $\langle \mathbf{3} \rangle$ What linear-algebraic property of the matrix field $g_{ij}(q)$ would ensure that the kinetic energy is non-negative in *any* state of the system? Define the linear algebraic property and explain using Dirac bra-ket notation.