Classical Mechanics (PG), Autumn 2013 CMI
Problem set 17
Due at the beginning of lecture on Monday Nov 18, 2013
Double pendulum, oscillations

1. $\langle \mathbf{9} \rangle$ Consider the double pendulum in the *small oscillation* approximation.

   (a) $\langle \mathbf{2} \rangle$ Identify the Riemannian metric g_{ij} on the torus $S^1 \times S^1$ defined by the kinetic energy $T = \frac{1}{2} g_{ij} \dot{\theta}^i \dot{\theta}^j$ in the small oscillation approximation.

   (b) $\langle \mathbf{3} \rangle$ Is it a flat/curved metric? Find out by calculating the Riemann tensor.

   (c) $\langle \mathbf{4} \rangle$ Is the picture of this torus as the surface of a Vadai/tyre-tube embedded in 3d Euclidean space $\mathbb{R}^3$, a geometrically faithful (i.e. isometric) depiction?

2. $\langle \mathbf{7} \rangle$ Consider the double pendulum in the **small oscillations** approximation $|\theta_i| \ll 1$, with hamiltonian

$$H(\theta_i, p_i) = \frac{1}{2ml^2} \left[p_1^2 + 2p_2^2 - 2p_1 p_2 \right] + mgl \left[\theta_1^2 + \frac{\theta_2^2}{2} - 3 \right]. \quad (1)$$

   Show that a constant energy $H = E$ hypersurface is contained in a finite region of phase space. Why is this intuitively/physically expected?

3. $\langle \mathbf{5} \rangle$ Recall the time-independent Schrödinger eigenvalue problem for a particle moving along a line in a potential $V(x)$

$$-\frac{\hbar^2}{2m} \psi''(x) + V(x)\psi(x) = E\psi(x) \quad (2)$$

   Formulate this equation as a pair of first order ODEs and write it in matrix form $x' = Ax$. Identify the matrix of coefficients A and show that it is traceless but not symmetric in general.