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An interacting parton model for quark and anti-quark distributions in the baryon

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Abstract

In this paper we study a $1+1$ dimensional relativistic parton model for the structure of baryons. The quarks and anti-quarks interact through a linear potential. We obtain an analytic formula for the isospin averaged valence quark distribution in the chiral and large N_c limits. The leading $\frac{1}{N_c}$ and non-zero current quark mass corrections are estimated. Then we extend this model to include 'sea' and anti-quarks. We find that the anti-quark content is small at a low value of Q^2. Using these distributions as initial conditions for Q^2 evolution, we compare with experimental measurements of the structure function $xF_3(x,Q^2)$ and find reasonable agreement. The only parameters we can adjust are the fraction of baryon momentum carried by valence quarks and the initial scale Q_0^2. © 2000 Published by Elsevier Science B.V.

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1. Introduction

We present a variational parton model description for the structure of baryons as measured in Deep Inelastic Scattering. This model enables us to calculate the x_B dependence of the structure function xF_3 at an initial value of Q_0^2. We compare this prediction with experimental measurements by CCFR and CDHS collaborations.

In Deep Inelastic Scattering, the longitudinal momenta of the partons dominates their transverse momenta. Indeed, as pointed out by Altarelli, Parisi and others [1], a perturbative treatment of transverse momenta, with an upper cut-off Q, leads to the same scaling violations as predicted by the Operator Product Expansion in the leading logarithmic approximation. By the uncertainty principle, the virtual photon momentum Q, is a measure of the size of transverse momenta being probed.

While the Q^2 dependence (for sufficiently large Q^2) of the structure functions is well understood [1,2], the x_B dependence is harder to understand since it deals with the formation of a relativistic bound state. We make the following ansatz: At some low value of $Q^2 = Q_0^2$, the transverse momenta of the partons may be ignored as a first approximation.

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The x_B dependence of quark and anti-quark distributions at Q_0^2, is then determined by solving a $1+1$ dimensional model. In this model, quarks interact via a linear potential in the null coordinates. This is the simplest potential consistent with Lorentz covariance.

We also suppose that the valence quarks carry a fraction f of the total baryon momentum at $Q^2 = Q_0^2$. The rest being carried by gluons, anti-quarks and sea-quarks. In a previous numerical study [3], we determined the valence quark distribution in this interacting parton model. Here, we extend this model to include anti-quarks and also obtain analytic formulae for the valence quark distributions, within a variational approach. Based on the two parameters Q_0 and f, we predict the initial x_B dependence of the iso-spin averaged quark and anti-quark distributions. Finally, to compare with experimental data at higher Q^2, we use the solutions of this model as initial conditions for the DGLAP evolution equations.

We do not derive this model from a more basic theory. It is proposed merely as a phenomenological parton model for an approximate description of Deep Inelastic Scattering.

Let us now give a brief introduction to our analysis of the interacting parton model. The partons are assumed to be relativistic particles interacting through a linear potential. The number of colors N_c is kept variable and we work mostly in the limit of a large number if colors. To simplify this many body problem we ignored the anti-quark degrees of freedom in [3]. The baryon wave function was determined by the principle that it minimizes the $(mass)^2$ of the baryon. Within a Hartree approximation, the valence quark wavefunction is the solution of a non-linear integral equation which was solved numerically in [3].

In this letter we first show that the true minimum of $(mass)^2$ occurs for a configuration that consists only of valence quarks, in the chiral and large N_c limits. The deviation of the anti-quark distribution from zero is measured by the dimensionless parameter $\frac{m^2}{\tilde{g}^2}$ in addition to $\frac{1}{N_c}$ corrections, which we estimate. Here m is the current quark mass and $\tilde{g}$ a coupling constant.

In order to determine the anti-quark content of the baryon, we perform a unitary transformation on the Fermionic Fock space, starting from a purely valence state. This transformation is like a Bogoliubov transformation which mixes positive and negative momentum states. It is sufficient to consider Bogoliubov transformations parametrized by a single angle θ which is determined by minimizing the $(mass)^2$ of the baryon. We find that in the large N_c limit, θ vanishes for zero current quark mass. For physically reasonable values of $\frac{m^2}{\tilde{g}^2}$, the anti-quarks carry less than a percent of baryon momentum. This is at an initial value of $Q^2 = Q_0^2$, at which transverse momenta are neglected. The isospin averaged quark and anti-quark distributions are found within a variational approximation.

To compare our results with experimental data, we evolve the distributions to higher values of Q^2 via the DGLAP equations [1]. However, the gluon distributions, which we have not determined, appear in the evolution equations. It turns out that the difference between quark and anti-quark distributions (valence quark distribution: $q^V(x_B,Q^2) = \sum_{\beta=u,d}(q^\beta(x_B,Q^2) - q^{\bar{\beta}}(x_B,Q^2))$) evolves independently of the gluon distribution to leading order. Moreover, if we ignore certain correlations, this difference is the average of the structure function F_3 measured in neutrino and anti-neutrino Deep Inelastic Scattering [2]. Given the x_B dependence of the parton distribution functions (PDFs) at an initial Q_0^2, their Q^2 evolution is determined by the DGLAP equation with the splitting function P_{qq} calculated perturbatively:

$$\frac{dq^V(x_B,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_{x_B}^1 \frac{dy}{y} q^V(y,t) P_{qq}\left(\frac{x_B}{y}\right).$$

Here $t = \log(Q^2/Q_0^2)$. The normalization $\nu(Q_0^2) = \int_0^1 dx_B q^V(x_B,Q_0^2)$ is determined by integrating the DGLAP equation with initial condition $\nu(\infty) = N_c = 3$ from $Q^2 = \infty$ to Q_0^2. Due to the large Q^2 range involved, we determine $\nu(Q_0^2)$ to high order. Within our approximations, $\nu(Q_0^2) = \int_0^1 dx_B F_3(x_B,Q_0^2)$, which is given by the GLS sum rule [4]. If we denote the isospin averaged valence quark probability density as $V(x_B,Q_0^2)$, then $q^V(x_B,Q_0^2) = \nu(Q_0^2)\, V(x_B,Q_0^2)$. In section 4 we compare our predictions for $xF_3(x,Q^2)$ with experimental measurements by the CDHS and CCFR collaborations. Our predictions agree well with data for a choice of parameters $f=\frac{1}{2}$ and $Q_0^2=0.4\ \mathrm{GeV}^2$ for which $\nu(Q_0^2)=2.25$. This choice of parameters is consistent with phenomenological fits to data [2,5]. However, it would be useful to know what the 'best-fit' values of these parameters are.

An impressive discretized light-cone (DLCQ) analysis of 2 dimensional QCD was done by Hornbostel et al. [6]. Our phenomenological parton model provides a complementary physical approach to their more direct numerical diagonalization of the hamiltonian. The 2-dimensional valence quark wave function we find reduces precisely to the one obtained in [6] when we set $f=1$. We do not find a similar concordance with the more conventional lattice QCD methods: the lightcone methods seem to incorporate the physical phenomena much more directly. Other DLCQ [7] calculations study the meson and glueball spectra of 2d models. We focus on the baryon. For other approaches see for instance [8].

2. Valence parton model

Let us begin by reviewing the valence quark approximation. Ref. [3] may be consulted for details. We assume that the momenta of the partons in the x^1 direction are large compared to the transverse momenta; which we ignore. We use null co-ordinates[1] where the null momentum $p=p_0-p_1$ is the basic kinematic variable. Then the kinetic energy of a free particle of mass m is $p_0=\frac{1}{2}(p+\frac{m^2}{p})$. So the wave function of a quark will vanish for negative p while that of an anti-quark vanishes for positive p.

If we ignore anti-quarks as in [3], then the baryon wavefunction $\tilde{\psi}(\{\nu_i,\alpha_i,p_i\})$ depends on the colors, flavours $(\alpha_i=1,\cdots,M)$ and null momenta of the N_c valence quarks. $\tilde{\psi}(\{\nu_i,\alpha_i,p_i\})=\epsilon_{\nu_1,\cdots,\nu_{N_c}}\tilde{\psi}(\{\alpha_i,p_i\})$ since the baryon is a color singlet. Moreover, since the null momenta are positive, the sum of quark momenta cannot exceed the total baryon momentum P. In particular, the wave function must vanish for $p_i>P$. Since the ϵ tensor is anti-symmetric in color, the wavefunction must be symmetric in the remaining variables: partons behave like bosons in the momentum, spin and flavour variables. The ground state wave function is determined by minimizing the total energy

$$\mathscr{E}_{N_c}[\tilde{\psi}]=\sum_{\alpha_1\cdots\alpha_{N_c}}\int_0^P\sum_{i=1}^{N_c}\frac{1}{2}\left[p_i+\frac{\mu_{\alpha_i}^2}{p_i}\right]\times|\tilde{\psi}(\alpha_1,p_1;\cdots\alpha_{N_c},p_{N_c})|^2\times\frac{dp_1\cdots dp_{N_c}}{(2\pi)^{N_c}}+\frac{1}{2}g^2\sum_{\alpha_1\cdots\alpha_{N_c}}\int_{-\infty}^{\infty}\sum_{i\neq j}v(x_i-x_j)\times|\psi(\alpha_1,x_1;\cdots\alpha_{N_c},x_{N_c})|^2\times dx_1\cdots dx_{N_c}.$$

Here $\mu_{\alpha_i}^2=m_{\alpha_i}^2-\tilde{g}^2/\pi$ is the effective mass of the parton [3], which avoids a potential infrared divergence in the potential energy. Also, $\tilde{g}^2=g^2/N_c$. The simplest potential consistent with Lorentz invariance is linear, $v(x)=|x|/2$, which is also favoured by phenomenology [11]. $\tilde{g}$ is a coupling constant with the dimensions of mass. Our predictions turn out to be independent of $\tilde{g}$ in the chiral limit. In the ground state, we expect the Hartree ansatz

$$\tilde{\psi}(\{\alpha_j,p_j\})=2\pi\delta\left(P-\sum_i p_i\right)\prod_{i=1}^{N_c}\tilde{\psi}(\alpha_i,p_i)$$

to be a good approximation. The valence quark wave function is normalized to have unit length, $\sum_{\alpha=1}^M\int_0^P|\tilde{\psi}(\alpha,p)|^2\frac{dp}{2\pi}=1$. It must also satisfy the momentum sum rule:

$$N_c\sum_\alpha\int_0^P p|\tilde{\psi}(p)|^2\frac{dp}{2\pi}=fP.$$

f here is the fraction of baryon momentum carried by the valence quarks, which is roughly a half at low Q^2 [2]. In these two formulae, we are ignoring correlations that are suppressed for large-N_c. They differ from the exact formulae in the same way as

[1] See Appendix to [9] for kinematics.

the canonical ensemble differs from the micro-canonical ensemble in statistical mechanics.

Since we are interested in the isospin averaged distributions, we will average over spin-flavour degrees of freedom. Thus we look for a wave function that is non-zero only for a single value of α: $\tilde{\psi}(\alpha,p)=\delta_{\alpha,1}\tilde{\psi}(p)$.

In second quantized language, this corresponds to the valence state $|V\rangle = a_{\tilde{\psi}}^{1\dagger}\cdots a_{\tilde{\psi}}^{N_c\dagger}|0\rangle$. Here, $a_{\tilde{\psi}}^{j\dagger}$ creates a quark with color j in the state $\tilde{\psi}$. These operators satisfy canonical anti-commutation relations (CAR): $\{a_{iu},a_{v}^{j\dagger}\}=\delta_i^j\langle u,v\rangle$ with respect to the Dirac vacuum $|0\rangle$ where all negative energy states are filled and positive ones empty: $a_{\tilde{\psi}_-}^{i\dagger}|0\rangle=0$ and $a_{j\tilde{\psi}_+}|0\rangle=0$. $\tilde{\psi}_-(p)$ vanishes for $p\geq 0$ and $\tilde{\psi}_+(p)$ for $p\leq 0$. The Pauli principle requires that the density matrix $\tilde{\rho}_V(p,q)=\langle V|\frac{1}{N_c}\hat{a}^{i\dagger}(p)\hat{a}_i(q)|V\rangle$ is a hermitean projection operator: $\int_{-\infty}^{\infty}\tilde{\rho}_V(p,r)\tilde{\rho}_V(r,q)\frac{dr}{2\pi}=\tilde{\rho}_V(p,q)$. The eigenvalues of the density matrix are the occupation numbers of particles, for a projection operator these are 0 or 1 as required by the Pauli principle. For a state containing one baryon, the normal ordered trace of the density matrix is equal to one: $tr(\tilde{\rho}_V(p,q)+\frac{1}{2}\tilde{\delta}(p,q)(\mathrm{sgn}(p)-1))=1$ by a use of the CAR. $\tilde{\delta}(p,q)$ is the identity matrix. The above Hartree ansatz, $\tilde{\rho}_V(p,q)=\tilde{\psi}(p)\tilde{\psi}^*(q)+\frac{1}{2}\tilde{\delta}(p,q)(1-\mathrm{sgn}\,p)$ satisfies these constraints.

A Lorentz invariant formulation is to minimize the mass $\mathscr{M}/N_c$ of the Baryon per quark:

$$\frac{\mathscr{M}^2}{\tilde{g}^2N_c^2}=\left[\frac{1}{2}\int_0^P p|\tilde{\psi}(p)|^2\frac{dp}{2\pi}\right]$$
$$*\left[\frac{1}{2}\int_0^P\frac{1}{2p}|\tilde{\psi}(p)|^2\left(\frac{m^2}{\tilde{g}^2}-\frac{1}{\pi}\right)\frac{dp}{2\pi}\right.$$
$$\left.+\frac{1}{2}\int_{-\infty}^{\infty}dxdy|\psi(x)|^2|\psi(y)|^2\frac{1}{2}|x-y|\right].$$

2.1. Analytic results in the large N_c limit

In [3] we solved the integral equation for the minimization of energy numerically. There is in fact an analytic solution in the chiral $(m^2/\tilde{g}^2\to 0)$ and large N_c limits. The boundary condition is that $\tilde{\psi}(p)$ must vanish for $p>P$. However, P is an extensive variable, $P\sim N_c$. So for $N_c=\infty$, the valence quark wave function is not required to vanish for any finite value of p. If we use the intensive quantity $\bar{P}=P/N_c$, the analog of momentum fraction is $\bar{x}_B=p/\bar{P}$, but the wave function is not required to vanish beyond $\bar{x}_B=1$. In order to compare directly with a wave function computed for $N_c=3$, we pick the reference frame in which $\bar{P}=\frac{1}{3}$. The momentum sum rule then becomes

$$\int_0^\infty p|\tilde{\psi}(p)|^2\frac{dp}{2\pi}=f\bar{P}.$$

It can be checked explicitly that

$$\tilde{\psi}(p)=\sqrt{\frac{2\pi}{f\bar{P}}}\,\mathrm{e}^{-p/2f\bar{P}}$$

is an exact solution to the integral equation for the minimization of baryon $(mass)^2$. Alternatively, we can calculate $\mathscr{M}^2/N_c^2$ for this wavefunction and see that it is zero. The potential and self energies cancel each other. Thus, in this limit the minimum of $(mass)^2$ actually occurs for a purely valence quark configuration. Since $\tilde{g}\sim\Lambda_{\mathrm{QCD}}\sim 200$ MeV and m_u, $m_d\sim 5$–8 MeV, this should be a good approximation provided the $1/N_c$ corrections are small. This is indeed the case, as we show below. For $f=\frac{1}{2}$, the valence quark density normalized to one is $V(\bar{x}_B)=6\mathrm{e}^{-6\bar{x}_B}$.

2.2. Leading order $1/N_c$ correction

The leading order effect of finite N_c is to restrict the range of quark momenta to $p<P$. Now, $(1-\frac{p}{n})^n\to\mathrm{e}^{-p}$ as $n\to\infty$. Therefore, $\tilde{\psi}(p)=Cp^a(1-\frac{p}{P})^b$, $0\leq p\leq P$ should be a good ansatz for the ground state wave function for finite N_c. C is determined by normalization. The momentum sum rule implies that $b=\frac{N_c}{2f}-1+a(\frac{N_c}{f}-1)$. The minimiza-

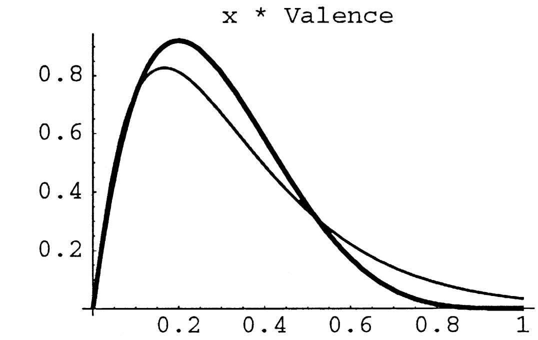

Fig. 1. Comparison of valence quark distribution $\bar{x}_B q^V(\bar{x}_B, Q_0^2)$ obtained in the large-N_c limit (thin curve) with the variational estimate $x_B q^V(x_B, Q_0^2)$ after taking into account the leading $\frac{1}{N_c}$ correction (thick curve).

tion of energy implies that a satisfies the transcendental equation

$$\frac{\pi m^2}{\tilde{g}^2} = 1 + \int_0^1 \frac{dy}{y^2}\left[(1+y)^a + (1-y)^a - 2\right] + \int_1^\infty \frac{dy}{y^2}\left[(1+y)^a - 2\right],$$

which we derived in [3]. In the limit of chiral symmetry, $a \to 0$. If valence quarks carry all the momentum of the baryon, $f = 1$, and $V(x_B) = (N_c - 1)(1 - x_B)^{N_c - 2}$, which is identical to the result obtained from DLCQ, reported in [6]. However, valence quarks carry only about half the baryon momentum, so that for $N_c = 3$ our variational estimate for the valence quark density is $V(x_B, Q_0^2) = 5(1 - x_B)^4$; this agrees well with our numerical solution from Ref. [3].

It is evident from Fig. 1 that the primary effect of the $\frac{1}{N_c}$ correction is to make the distribution vanish beyond $\frac{p}{P} = 1$. The actual shape of the distribution is already well captured by our analytic solution in the large-N_c limit.

3. Extension of parton model to include anti-quarks

Now we turn to the anti-quark content of the baryon. As we will see below, the minimum of Baryon $(mass)^2$ occurs for a small anti-quark content when the parameter $m^2/\tilde{g}^2$ is small. In order to determine it, we need to allow for states with negative momenta. However, only the difference between the quark and anti-quark numbers is conserved in the full theory. Therefore, the baryon must be in a linear superposition of states containing η anti-quarks and $N_c + \eta$ quarks, for $\eta = 0, 1, \cdots, \infty$. The energy of a state containing η anti-quarks is

$$\sum_{\nu_j}\sum_{i=1}^{N+2\eta}\int_{-P}^{P}\frac{1}{2}\left(p_i + \frac{\mu^2}{p_i}\right)|\tilde{\psi}_\eta(\{\nu_j, p_j\})|^2 \Pi_k\, dp_k + \frac{g^2}{2}\sum_{\nu_k}\sum_{i\neq j}\int_{-\infty}^{\infty}\tfrac{1}{2}|x_i - x_j||\psi_\eta(\{\nu_k, x_k\})|^2 \Pi_m\, dx_m,$$

where $\tilde{\psi}_\eta(\{\nu_i, p_i\})$ is the wave function of such a state. $\{\nu_i, p_i\}_{i=1}^{N_c+\eta}$ refer to the colors and momenta of the quarks, while $\{\nu_i, p_i\}_{i=N_c+\eta+1}^{N_c+2\eta}$ refer to anti-quarks. The wave function vanishes for negative quark momenta and positive anti-quark momenta. The total energy is the sum of the energies for each value of η.

Now we shall argue that the *ground state* of the baryon is to a good approximation determined by three orthonormal one-parton states ψ and ψ_+ which describe quarks and ψ_- which describes anti-quarks. We will continue to work in a factorized Hartree approximation, ignoring correlations except when they are required by the Pauli principle or color invariance.

In the absence of anti-quarks, there are just N_c quarks whose wavefunction is completely anti-symmetric in color. Within the Hartree approximation, they all occupy the same single parton positive momentum state ψ, which minimizes the $(mass)^2$ of the baryon. Now consider adding a quark and an anti-quark $(\eta = 1)$. The anti-quark will occupy the negative momentum state ψ_- that minimizes the energy. Suppose all the $N_c + 1$ quarks occupy the state ψ. Then, by the Pauli exclusion principle, the color part of the quark wave function must be totally anti-symmetric. However, there is no completely anti-symmetric tensor in $N_c + 1$ indices transforming under $SU(N_c)$. Therefore, we are forced to introduce a new positive momentum state ψ_+, that must be orthogonal to the filled state ψ. In order that this cost a minimal amount of energy, we expect $\tilde{\psi}_+(p)$ to have only one more node than $\tilde{\psi}(p)$. As long as $\eta \le N_c$, in the ground state, the quarks and anti-quarks occupy the states ψ, ψ_+, ψ_-. If $\eta > N_c$, we would have to introduce another pair of orthonormal states. However, we find that these additional corrections are very small.

Now we shall argue that the configuration containing valence, sea and anti-quarks (say $|\mathrm{VSA}\rangle$) can be obtained by a unitary transformation acting on the valence quark state $|V\rangle$: a Bogoliubov transformation. It must be unitary in order that $\langle \mathrm{VSA}|\mathrm{VSA}\rangle = 1$. The operator that creates a quark in state ψ_+ and an antiquark in state ψ_- is $a_{j\psi_-} a^{j\dagger}_{\psi_+}$; we sum over color indices to produce a color invariant state. Thus the unitary transformation we seek is the identity except in the two dimensional subspace spanned by ψ_+ and ψ_-. Thus our variational ansatz is $|\mathrm{VSA}\rangle = \mathrm{e}^{\theta[a_{j\psi_-} a^{j\dagger}_{\psi_+} - \mathrm{h.c.}]}|V\rangle$. The density matrix of quarks in the new state can now be calculated:

$$\begin{aligned}\tilde{\rho}_{\mathrm{VSA}}(p,q) &= \tilde{\psi}(p)\tilde{\psi}(q) - \sin^2\theta\left[\tilde{\psi}_-(p)\tilde{\psi}_-(q)\right. \\ &\quad \left. - \tilde{\psi}_+(p)\tilde{\psi}_+(q)\right] - \tfrac{1}{2}\sin 2\theta\left[\tilde{\psi}_-(p)\tilde{\psi}_+(q)\right. \\ &\quad \left. + \tilde{\psi}_+(p)\tilde{\psi}_-(q)\right] + \tfrac{1}{2}\tilde{\delta}(p,q)(1 - \mathrm{sgn}\, p).\end{aligned}$$

Physical quantities are expressed most simply in terms of the normal ordered density matrix: $\tilde{M}(p,q) = -2\tilde{\rho}(p,q) + \tilde{\delta}(p,q)(1 - \mathrm{sgn}\, p)$. For example, the baryon number density in momentum space is

$$\begin{aligned}&\tilde{M}(p,p) - \tilde{M}(-p,-p) \\ &\quad = |\tilde{\psi}(p)|^2 + \sin^2\theta\left[|\tilde{\psi}_+(p)|^2 - |\tilde{\psi}_-(-p)|^2\right].\end{aligned}$$

This confirms the interpretation of ψ_- as the anti-quark wavefunction. The $(mass)^2$ is given by

$$\begin{aligned}\frac{\mathscr{M}^2}{N_c^2} &= \left[-\frac{1}{2}\int_{-P}^{P} p\tilde{M}(p,p)\frac{dp}{2\pi}\right] \\ &\quad * \left[-\frac{1}{2}\int_{-P}^{P}\tilde{M}(p,p)\frac{\mu^2}{2p}\frac{dp}{2\pi}\right. \\ &\quad \left. + \frac{\tilde{g}^2}{8}\int_{-\infty}^{\infty} dx dy |M(x,y)|^2 \tfrac{1}{2}|x-y|\right].\end{aligned}$$

The variational quantities ψ, ψ_+, ψ_-, and θ are determined by minimizing the baryon $(mass)^2$. In the ground state, we expect $\tilde{\psi}(p)$ and $\tilde{\psi}_-(p)$ to have no nodes (except possibly at the boundaries $p = 0, P$), while $\tilde{\psi}_+$ must have one more node. We estimate them variationally.

3.1. Large N_c analysis

Working in the $N_c \to \infty$ limit, the form of the analytic solution suggests the choice

$$\begin{aligned}\tilde{\psi}(p) &= C\left(\frac{p}{\tilde{g}}\right)^a \mathrm{e}^{-b\frac{p}{\tilde{g}}}, \tilde{\psi}_+(p) \\ &= C_+\left(\frac{p}{\tilde{g}}\right)^a\left[\frac{p}{\tilde{g}} - C_1\right]\mathrm{e}^{-b\frac{p}{\tilde{g}}}\end{aligned}$$

for $p > 0$ and $\tilde{\psi}_-(p) = \tilde{\psi}(-p)$ for $p < 0$. (For other ranges of p these functions must vanish.) C_1 is determined by the orthogonality condition while C, C_+ are fixed by the normalization conditions. The variational parameter b determines the reference frame. The Lorentz invariant quantity $\mathscr{M}^2$ is independent of b. Thus the variational principle will determine a and θ and hence the wavefunctions. The actual minimization of $\mathscr{M}^2$ is a lengthy but straightforward calculation. Most of the energy integrals can be evaluated analytically and we do them using the symbolic package Mathematica. We find that $\theta, a \to 0$ as $\frac{m^2}{\tilde{g}^2} \to 0$ recovering the purely valence exponential solution. For a physically reasonable value of $m^2/\tilde{g}^2 \sim m_{u,d}^2/\Lambda_{\mathrm{QCD}}^2 \sim 0.001$, we estimate $\theta = 0.02$ and $a = 0.035$. This corresponds to a small but non-vanishing anti-quark content in the baryon.

3.2. Leading order $1/N_c$ correction

As in the valence quark case, the leading $1/N_c$ effect is to make these wave functions vanish beyond $p = P$. We estimate this correction using the ansatz

$\tilde{\psi}(p) = Dp^a(1-p)^b, \tilde{\psi}_+(p) = D_+ p^a(p - D_1)(1-p)^b$ for $1 \geq p \geq 0$ and $\tilde{\psi}_-(p) = \tilde{\psi}(-p)$ for $-1 \leq p \leq 0$. Here $P = 1$ and for other ranges of p these functions must vanish. D_1 is determined by the orthogonality condition while D, D_+ are fixed by the normalization conditions. For the choice $Q_0^2 = 0.4\,\mathrm{GeV}^2$, $f = \frac{1}{2}$, $m^2/\tilde{g}^2 \sim .001$, we get $\theta = 0.02$, $a = 0.035$, $b = 2.175$ for the variational parameters. The valence quark distribution is normalized to $\nu(Q_0^2) = 2.25$ while the normalization of the anti-quark distribution is determined as a consequence to be $\nu(Q_0^2)\sin^2\theta$. Since $\sin^2\theta \sim 10^{-4}$, the primordial anti-quarks carry only about .01% of the baryon momentum.

These results are identical to what we obtained from a more field theoretic point of view in [9,10]. They also agree with the DLCQ analysis of [6] as pointed out in Section 2.2. However, the parton model point of view presented here is much simpler. Moreover, the GRV collaboration [5] have obtained a reasonably good fit to Deep Inelastic Data for $x_B > 10^{-2}$ starting with a vanishing anti-quark distribution at an initial $Q_0^2 \sim 0.2$ GeV^2. Our approximations are not expected to be valid for extremely low values of the momentum fraction, where the assumption that longitudinal momenta dominate, becomes questionable.

Thus we find that the valence quark picture is quite accurate: the 'primordial' anti-quark distribution is very small. The anti-quark content is zero not only in the non-relativistic limit $m \gg \tilde{g}$ but also (somewhat surprisingly) in the chiral limit $m = 0$ when $N_c \to \infty$, with $\frac{1}{N_c}$ corrections being small. Nevertheless, a substantial anti-quark content is generated by Q^2 evolution.

4. Comparison with experimental data

Finally, we compare with experimental data at higher values of Q^2. The DGLAP Q^2 evolution equation is integrated numerically. We set $N_c = 3$, $\Lambda_{\mathrm{QCD}} = 200$ MeV and the current quark mass $m = 0$. The parameters f, Q_0^2 should be determined by a best fit to experimental data. For now we assign to them reasonable values $f = \frac{1}{2}$ and $Q_0^2 = 0.4$ GeV^2 which are consistent with phenomenological fits to data. From the GLS sum rule we get $\nu(0.4\ \mathrm{GeV}^2) = 2.25$ [4]. In Fig. 2, we show a comparison with xF_3 measurements by the CDHS and CCFR collaborations [12] at $Q^2 \sim 13$ GeV^2. The small range $0.4 \leq Q^2 \leq 13$ GeV^2 over which we are evolving justifies the use of the leading order DGLAP equation. The plot shows that our prediction agrees reasonably well with the experimental measurements.

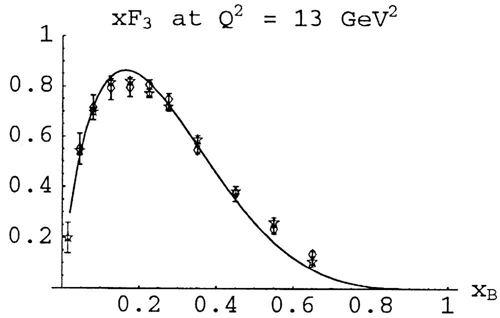

Fig. 2. Comparison of predicted xF_3 at $Q^2 = 13$ GeV^2 (solid curve) with measurements by CCFR (★) at 12.6 GeV^2 and CDHS (♦) at $12.05 \leq Q^2 \leq 14.3$ GeV^2. $Q_0^2 = 0.4$ GeV^2 and $f = \frac{1}{2}$.

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