

Texts and Readings in Physical Sciences

Volume 22

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Govind S. Krishnaswami

Classical Mechanics

From Particles to Continua and Regularity to
Chaos

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ISSN 2366-8849 ISSN 2366-8857 (electronic)
Texts and Readings in Physical Sciences
ISBN 978-981-97-4475-6 ISBN 978-981-97-4476-3 (eBook)
<https://doi.org/10.1007/978-981-97-4476-3>

Jointly published with Hindustan Book Agency.
The print edition is not for sale in India. Customers from India please order the print book from: Hindustan Book Agency, P 19 Green Park Extension New Delhi 110016 India.
ISBN of the Co-Publisher's edition: 978-81-957829-4

Mathematics Subject Classification: 70Hxx, 70Fxx, 70Exx, 37N05, 35Qxx, 70Kxx, 76-XX, 70G45

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The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

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Preface

Why Classical Mechanics?

Classical¹ mechanics traces its origins to the study of the motion of bodies. It is an old subject whose basic principles were formulated in Galileo's and Newton's time (1600s). However, the subject remains alive and continues to thrive. This is because (a) it is the basis for the formulation of most physical systems and remains the predictive framework in which we try to understand and control the vast majority of physical phenomena despite the advent of the quantum theory,² (b) the theoretical edifice of mechanics has undergone repeated rejuvenation allowing it to be understood in increasingly deep ways (Lagrangians, Hamiltonians, Poisson brackets, symplectic forms, canonical transformations, Hamilton-Jacobi equation, perturbation theory, Dirac-Bergmann constraints, Lax pairs, Kolmogorov-Arnold-Moser (KAM) theory, symbolic dynamics, renormalization and so forth) and (c) old classical phenomena like turbulence are yet to be satisfactorily understood and new phenomena such as those surrounding integrable, chaotic, mixed and open systems pose fresh challenges. The history of classical mechanics also reveals that the theoretical and computational schemes developed to understand and solve Newton's equations of mechanics (such as conservation laws, variational principles, constraints, the Hamiltonian formulation or that in terms of geodesics or Hamilton-Jacobi wavefronts) have a utility beyond the validity of Newton's laws. They have even been used to discover and formulate other or more accurate laws of nature (such as those governing electromagnetism or the quantum dynamics of relativistic particles and fields) as well as to set up effective or statistical frameworks to deal with unexpected 'emergent' phenomena that arise in the presence of nonlinearities or when many degrees of freedom interact. Furthermore, developments in classical mechanics have been inspired by and used in numerous practical problems of science and engineering such as projectile motion, timekeeping,

¹ The adjective *classical* conveys that it is the oldest and best established branch of mechanics. Technically, the term *classical* is used to distinguish it from *quantum* mechanics.

² Even phenomena or systems that require a quantum mechanical treatment are often fruitfully first understood in some classical limit.

construction of roads, bridges, dams, musical instruments and gyroscopes, flight of airplanes, launching and placement of satellites and weather prediction. If we add to this the beauty of the mathematical form of classical dynamics, we begin to see why it remains a central part of physics research and education.

Scope of this Book

This book is meant primarily for postgraduate students of physics and their teachers. However, it should also be of use to undergraduates, researchers and others interested in mechanics, especially when viewed as a part of the larger fabric of theoretical physics. An attempt has been made to keep the book reasonably self-contained by developing topics from basic principles and by providing physical context, intuition and reasoning. Topics are presented in a manner that elucidates the thought process behind calculations. Mathematical constructions and terms are motivated and explained alongside physical developments. Readers should find them useful in thinking about physics and communicating effectively. It is hoped that these features will make the book suitable both for use in taught courses and self-study. Many details and subtle issues are clarified, which should enable teachers to explain ideas effectively. The process of learning mechanics is not linear: a novice is not expected to follow, at the first instance, every concept that is presented. Things tend to become clearer on subsequent readings. Footnotes are an integral part of this book and are used to avoid interrupting the flow. They contain qualifications, explanations, additional material, etc. While some of them are meant for instructors and advanced readers, many should be helpful even to first-time readers. Starred sections, on the other hand, contain topics that may be skipped on a first reading without affecting the continuity of the material. It is hoped that the reader will acquire a working knowledge of, affinity and appreciation for mechanics, its challenges and its connections to other areas of physics, as well as a degree of theoretical sophistication by working through this book.

Genesis of this Book

The specific suggestion to write this book for the *Texts and Readings in Physical Sciences* (TRiPS) series came from D. K. Jain of Hindustan Book Agency on 19 January, 2018, although he had mentioned it more casually earlier as well. He followed this up with periodic exhortations to start writing. The fact that both my guru S. G. Rajeev and my mama (maternal uncle) N. Mukunda (with E. C. G. Sudarshan) had written advanced books on mechanics [1] and [2] was a source of inspiration, but also made it a daunting task. Encouragement and feedback from students who took my courses, my colleagues H. S. Mani, K. P. N. Murthy, A. Laddha and K. G. Arun (who had seen my lecture notes) and D. Ghoshal helped convince me that it was

all right to attempt yet another book on mechanics. Although my lecture notes had been with me for a few years, it was not until the day after new year's day, 15 April, 2020 that I started preparing the manuscript. The lock-down due to the COVID-19 pandemic played an important role in helping me get started and sustaining the effort. I soon realized how inadequate my rough lecture notes were, but it took longer for me to recognize just how much effort was needed to write this book! The process itself has been fun and rewarding and I hope readers will enjoy this book.

Acknowledgements

The classic text by Landau and Lifshitz [3] remains a good place to learn mechanics. I have occasionally relied on their exposition. My interest in mechanics and the viewpoint in this book owe a lot to what I learned from S. G. Rajeev. His work and writings continue to inspire me. I have also benefited from my collaboration with A. Thyagaraja, who has helped shape my viewpoint on fluid mechanics. Over the past decade, I have given lectures at several places in India (Chennai Mathematical Institute (CMI), Science Academy Refresher Courses and Workshops, etc.). The questions and comments from those who attended my lectures were useful in the preparation of my notes. I thank V. V. Sreedhar, G. Rajasekaran, H. S. Mani, M. Lakshmanan, C. Rao, A. N. Kumara, S. Ananth, K. Indulekha, N. V. Suryanarayana, D. Jatkar, M. Ashefas and others for asking me to deliver these lectures. Comments and suggestions from R. Jagannathan, R. Parthasarathy, G. Date, T. R. Ramadas, R. Nityananda, A. V. Khare, M. V. N. Murthy, J. K. Bhattacharjee, A. Das, A. Arul and anonymous reviewers have enriched the book, as have those of the Editors R. Ramaswamy and D. Ghoshal of TRiPS. Advice and suggestions from the publishers D. K. Jain and J. K. Jain of Hindustan Book Agency and S. Ahmad of Springer Nature have helped me prepare this book for publication. I am also grateful to my colleagues and the staff at CMI for the pleasant working environment.

Special thanks go to Ph.D. students Sachin Phatak, Sonakshi Sachdev, Himalaya Senapati, T. R. Vishnu, Ankit Yadav and Pritish Sinha as well as summer student Shivangi Dhiman with whom I have discussed many of the topics and exercises in this book. Shivangi helped prepare several of the figures and parts of the Index for the first three chapters. Her detailed study of the first drafts of these chapters helped iron out some obscurities. Ankit too helped with many figures, numerical calculations in Chaps. 8, 13 and 15 and gave much useful feedback on a majority of the chapters. Himalaya's critical comments on practically all chapters have significantly improved the exposition. He also helped with getting the LaTeX typesetting software package to do useful things! Sects. 2.8 and 9.5 on the three-body problem are based on the article [4] written with Himalaya. Pritish's questions upon reading portions of Chaps. 1, 2, 3, 4, 19 and Appendix B have helped clarify some subtle concepts. The introduction to quaternions in Appendix B.14 is based on the article [5] coauthored with Sonakshi. The discussion of Lax pairs in Chap. 11 and Chap. 17 is based on expository articles [6] and [7] written with Vishnu, who also made useful comments on Chap. 7.

I would like to acknowledge generous financial support in the form of a Ramanujan Fellowship, a Core Research Grant and a MATRICS grant from the Science and Engineering Research Board (SERB) of the Department of Science and Technology, Government of India, at various stages in the preparation of this book.

Finally, this book was made possible by the support I have received from my family, especially Krishnaswami, Leela and Maithreyi. It is written for Shraddha and dedicated to my parents and to Profs. C. S. Seshadri and G. Rajasekaran who unfortunately passed away while I was writing it.

Prerequisites

Although most mathematical methods are developed along the way, we assume some familiarity with calculus, vectors and matrices. Some of this background is in Appendix A while more may be found in [8, 9, 10, 11, 12, 13, 14, 15]. Connections to electromagnetism, optics, thermodynamics, relativity, quantum and statistical mechanics are occasionally pointed out. For those unfamiliar with these subjects, such remarks may simply be ignored. Finally, it is hoped that a reader can begin reading several of the chapters without having studied many of the preceding ones.

Arrangement of Material

The order of presentation is based on certain guiding principles. (1) Increasing number of degrees of freedom: point particles (1D–3D), rigid bodies and then continuous media. (2) A historical path through the development of the formalism of classical mechanics, progressing from the ideas of Galileo and Newton through the Bernoullis, d’Alembert, Maupertuis, Euler, Lagrange, Hamilton, Poisson, Jacobi, Lie, Hill, Poincaré, Lyapunov, etc. (3) Small oscillations to large oscillations: linear to nonlinear equations of motion. (4) The idea that the study of concrete examples often leads to general principles/formalisms which then inform on the behavior of other model systems. (5) Configuration and phase spaces that are topologically trivial (e.g., Euclidean spaces) followed by those that are not (e.g., circles, spheres, cylinders). (6) Increasing mathematical sophistication. (7) Regular dynamics to chaotic dynamics.

Outline of Chapters

For those who would like to begin with some mathematical and kinematical preliminaries, Appendix A deals with vectors in Euclidean space, kinematics, circular motion, polar coordinates, Taylor series, vector calculus, linear algebra

and the Fourier transform. We begin our study of mechanics in Chap. 1 with the Newtonian formulation of the motion of a particle on a line and discuss some of the general phenomena that arise. The harmonic oscillator and pendulum are introduced as important examples of systems with one degree of freedom. The notions of oscillatory time period, time delay and inverse problems are introduced. Chapter 2 is devoted to Kepler's two-body central force problem which is used to motivate and illustrate the consequences of Newton's laws of mechanics and gravity in the context of planetary orbits. Replacing the gravitational force with the Coulomb force between charged particles, we derive Rutherford's differential cross section for the scattering of alpha particles by gold nuclei. The utility of Keplerian orbits goes beyond the two-body problem: we show how they turn up in the Euler and Lagrange solutions of the three-body problem! With this background at hand, we move (in Chap. 3) to the conceptual and formal development from Galileo's principle of relativity to the Newtonian, Euler-Lagrangian, Hamiltonian, Poisson bracket and Euler-Maupertuis formalisms of mechanics. Chapter 3 has been placed after Chaps. 1 and 2 in keeping with the adage that physics tends to be developed through the study of examples or model systems. General principles often emerge from the study of specific systems. A reader may of course go straight to Chap. 3 and return to the previous chapters when necessary. In Chap. 4, we point out the shortcomings of Newtonian mechanics in treating phenomena where speeds of particles or frames relative to an inertial frame are comparable to that of light and give a brief introduction to special relativistic mechanics. In Chap. 5, we return to nonrelativistic mechanics and adopt a dynamical systems viewpoint where we think of mechanical systems in terms of their associated vector fields on the state space. We then turn to a variety of examples in Chaps. 6, 7, 8, 9: small oscillations for one degree of freedom, (not necessarily small) oscillations of a simple pendulum and quartic anharmonic oscillator, rigid bodies and motion in noninertial frames (with applications to Foucault's pendulum and the restricted three-body problem). The formalisms developed in earlier chapters are put to use here, while new techniques (both exact and approximate) are also introduced. We then return to general structural aspects by discussing canonical transformations, angle-action variables and the Hamilton-Jacobi equation in Chaps. 10, 11, and 12, where we also meet Poincaré recurrence, Liouville integrability, KAM tori and Lax pairs. In Chap. 13, we go back to the study of small oscillations, this time around static and periodic solutions of systems with two or more degrees of freedom. Pendula coupled by a spring, a diatomic molecule and the double pendulum are used to illustrate normal modes, while the Kapitza pendulum provides a context to examine the stability of oscillations. In Chap. 14, we discuss several simple examples of bifurcations of vector fields, which illustrate how the qualitative behavior of a system can change as a control parameter is varied. This is followed in Chap. 15, by examples of discrete-time and continuous-time dynamical systems (standard and logistic maps, double pendulum and Lorenz oscillator) illustrating the passage from regular to chaotic motion. The next four chapters (Chaps. 16, 17, 18, 19) provide an introduction to continuum mechanics via d'Alembert's wave equation for vibrations of a stretched string, Fourier's equation for heat diffusion and Brownian motion and finally fluid mechanics. Appendix A discusses mathematical and kinematical

preliminaries while Appendix B contains an informal introduction to concepts from the theory of manifolds, tensors, differential geometry, groups and Lie algebras that we use in our treatment of mechanical systems. We end with a list of books that treat related topics, cited references to the literature and a detailed index. Readers are encouraged to attempt the end-of-chapter problems, which are an integral part of the book. Each chapter begins with an introductory section providing conceptual context, outlining topics to be treated and mentioning further developments.

Planning a Course

A first course on mechanics for undergraduates has been taught by starting with background on vectors, kinematics, polar coordinates and vector calculus (Appendix A) followed by the framework of Newtonian mechanics (Sects. 3.1, 3.2), Galileo's relativity principle and Newton's laws (Sect. 3.3), concepts of phase space, conservation laws and their illustration via collisions (Sect. 3.4), motion in one dimension (Sects. 1.1, 1.2), Kepler's laws and Newton's law of gravity (Sect. 2.1), an introduction to the harmonic oscillator and simple pendulum (Sects. 1.1, 6.1, 1.5), uniformly accelerated frames (Sect. 9.1) and the elements of special relativity (Sects. 4.1, 4.2, 4.3, 4.4, 4.5, 4.6). A second course on mechanics could cover Lagrangian mechanics (Sects. 3.5, 3.6, 3.7, 3.8, 3.9, 3.10), Hamiltonian mechanics (Sects. 3.14, 3.15), Poisson brackets (Sect. 3.21), canonical transformations (Sects. 10.1, 10.4, time permitting), the gravitational two-body problem (Sects. 2.2, 2.3), small oscillations and normal modes (Sect. 13.1), damped harmonic oscillations (Sect. 6.4, time permitting), rigid bodies (Sects. 8.1, 8.2, 8.3, 8.4, 8.5) and noninertial frames of reference (Sect. 9.2). The content of a Master's course would depend on student background. It may be based on a selection from the following topics, with some being assigned as reading projects: motion in one dimension (Sects. 1.1, 1.2, 1.5), the Kepler problem (Sects. 2.1, 2.2, 2.3, 2.4, 2.5), the formalism of Newtonian, Lagrangian and Hamiltonian mechanics (Sects. 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.12, 3.14, 3.15, 3.18, 3.19, 3.21, 3.22), driven oscillations (Sect. 6.8) or anharmonic oscillations (Sect. 7.1), rigid body dynamics (Sect. 8.6, 8.7, 8.9, 8.10, 8.11, 8.12), canonical transformations (Sects. 10.1, 10.2, 10.4, 10.5, 10.6, 10.7, 10.8, 10.10), angle-action variables (Sects. 11.1, 11.2, 11.3, 11.4) and the Hamilton-Jacobi equation (Sects. 12.1, 12.3, 12.4). A course on nonlinear dynamics could treat vector fields in one and two dimensions (Sects. 5.1, 5.2, 5.3, 5.4), planar vector fields arising from small oscillations (Sects. 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7), the pendulum (Sect. 7.1) or anharmonic oscillator (Sects. 7.4, 7.5), infinitesimal canonical transformations (Sect. 10.6), Liouville's theorem (Sect. 10.8), Poincaré recurrence (Sect. 10.9), angle-action variables (Sects. 11.1, 11.2, 11.3, 11.4), Liouville integrability and KAM tori (Sect. 11.7), normal modes (Sect. 13.2), stability of periodic solutions (Sect. 13.4), bifurcations of vector fields (Sects. 14.1, 14.2) and

chaos (Sects. 15.1, 15.4). An introductory continuum mechanics course (sans elasticity) may be based on the unstarred sections in Chaps. 16, 17, 18, 19. Mathematical supplements from Appendix B may be introduced when the need arises or in a course on mathematical methods of physics. In planning courses, instructors should feel free to omit passages based on student background and time available. Many chapters are self-contained, so teachers may treat topics in an order they are comfortable with. For instance, motion in noninertial frames could be discussed before rigid bodies, although the corotating frame of a rigid body is a nice example of a noninertial frame. Finally, I hope to maintain a list of corrections/clarifications and additional resources at the web address: <http://www.cmi.ac.in/~govind>.

Chennai, India
May 2024

Govind S. Krishnaswami

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About This Book

This well-rounded and self-contained treatment of classical mechanics strikes a balance between examples, concepts, phenomena and formalism. While addressed to graduate students and their teachers, the minimal prerequisites and ground covered should make it useful also to undergraduates and researchers. Starting with conceptual context, physical principles guide the development. Chapters are modular and the presentation is precise yet accessible, with numerous remarks, footnotes and problems enriching the learning experience. Essentials such as Galilean and Newtonian mechanics, the Kepler problem, Lagrangian and Hamiltonian mechanics, oscillations, rigid bodies and motion in noninertial frames lead up to discussions of canonical transformations, angle-action variables, Hamilton-Jacobi and linear stability theory. Bifurcations, nonlinear and chaotic dynamics as well as the wave, heat and fluid equations receive substantial coverage. Techniques from linear algebra, differential equations, manifolds, vector and tensor calculus, groups, Lie and Poisson algebras and symplectic and Riemannian geometry are gently introduced. A dynamical systems viewpoint pervades the presentation. A salient feature is that classical mechanics is viewed as part of the wider fabric of physics with connections to quantum, thermal, electromagnetic, optical and relativistic physics highlighted. Thus, this book will also be useful in allied areas and serve as a stepping stone for embarking on research.

Contents

1	Mechanical Systems with One Degree of Freedom	1
1.1	Newton's Laws and Qualitative Characterization of Motion on a Line	2
1.2	Time Period of Conservative Oscillations Between Turning Points	7
1.3*	Inverse Problem: Determination of Potential from Time Period	9
1.4*	Time Delay in Unbounded 'scattering' Trajectories	11
1.5	Simple Pendulum: Basic Properties and Small Oscillations	14
1.6	Problems	18
	References	22
2	Kepler's Gravitational Two-Body Problem	23
2.1	Inverse Problem: Universal Law of Gravity from Kepler's Laws	24
2.2	Direct Problem: Center of Mass and Relative Vectors and Conservation Laws	28
2.3	Planetary Orbits	31
2.4	Time Period of Elliptical Orbits	36
2.5*	LRL Eccentricity Vector and Relations Among Conserved Quantities	37
2.6*	Collision of Two Gravitating Point Masses: Collision Time and Universality	41
2.7*	Rutherford Scattering Cross Section	45
2.8*	The Three-body Problem: Euler and Lagrange Solutions	53
2.9	Problems	56
	References	59

3	Newtonian to Lagrangian and Hamiltonian Mechanics	61
3.1	Time, Space, Light, Simultaneity, Causality, Homogeneity and Isotropy	61
3.2	Degrees of Freedom and Instantaneous Configurations	62
3.3	Newton's Laws and Galileo's Relativity and Equivalence Principles	63
3.4	Phase Space, Dynamical Variables, Conserved Quantities and Collisions	72
3.5	Principle of Extremal Action and Euler-Lagrange Equations	75
3.6	Nonuniqueness of Lagrangian	81
3.7	Conjugate Momenta, their Geometric Meaning and Cyclic Coordinates	81
3.8	Coordinate Invariance of the Form of Lagrange's Equations	83
3.9	Hamiltonian and Its Conservation	85
3.10	Symmetries to Conserved Quantities: Noether's Theorem	87
3.11*	Noether's Theorem When Lagrangian Changes by a Time Derivative	91
3.12	Inertial Frames of Reference and Galilean Invariance	92
3.13*	Polar Vectors, Axial Vectors, True Scalars and Pseudoscalars	97
3.14	Hamilton's Equations	99
3.15	Legendre Transform: Hamiltonian from Lagrangian	101
3.16*	Lagrange Multipliers and Constrained Extremization	104
3.17*	Singular Lagrangians and Constraints	106
3.18*	Action as a Function Along a Trajectory	109
3.19	Variational Principles for Hamilton's Equations	111
3.20*	Coordinate Invariance of Lagrange and Hamilton Equations	113
3.21	Canonical Poisson Brackets	115
3.22	Properties of the Poisson Bracket	117
3.23*	Canonical Formulation of Charged Particle in Electromagnetic Field	121
3.24*	Poisson Algebra of Conserved Quantities in the Kepler Problem	123
3.25*	Functional Independence of Conserved Quantities	126
3.26*	Noncanonical Poisson Brackets, Poisson and Symplectic Manifolds	131
3.27*	Free Particle Trajectories as Geodesics on Configuration Space	138
3.28*	Euler-Maupertuis Principle and the Jacobi-Maupertuis Metric	142
3.29	Problems	145
	References	161

4	Introduction to Special Relativistic Mechanics	163
4.1	Difficulties with Newtonian Mechanics	163
4.2	Postulates of Special Relativity	164
4.3	Synchronization of Clocks and Simultaneity	166
4.4	Lorentz Transformations	168
4.5	Time Dilation, Length Contraction, Proper Length and Time	171
4.6	Space-Like, Time-Like and Light-Like Intervals and Causality	173
4.7	Relativistic Addition of Velocities	174
4.8	Relativistic Momentum from Two Particle Collision	175
4.9	Relativistic Energy and Energy-Momentum Dispersion Relation	177
4.10*	Minkowski Space-Time and Relativistic Dynamics	179
4.11	Problems	185
	References	187
5	Dynamics Viewed as a Vector Field on State Space	189
5.1	Vector Fields from Newtonian and Hamiltonian Dynamics	190
5.2*	Vector Fields in One Dimension	194
5.3	Existence and Uniqueness of Solutions	199
5.4*	Vector Fields on the Phase Plane	203
5.5	Problems	211
	References	214
6	Small Oscillations for One Degree of Freedom	215
6.1	Linear Harmonic Oscillator in 1D and Neutral Stability	215
6.2	Linear Vector Fields on the Phase Plane	218
6.3	Phase Portrait from Spectrum of Coefficient Matrix	220
6.4	Damped Harmonic Oscillator: View from the Phase Plane	222
6.5	Critically Damped Oscillator: Deficient Coefficient Matrix	226
6.6*	Trace-Determinant Classification of Linear Fixed Points	228
6.7*	Robustness of the Linear Theory	230
6.8	Driven or Forced Oscillations	232
6.9*	Driven Damped Oscillations	239
6.10	Parametric Oscillations and Resonant Amplification	240
6.11	Problems	242
	References	248

7	Nonlinear Oscillations: Pendulum and Anharmonic Oscillator	249
7.1	Simple Pendulum: View from Phase Space	249
7.2*	Introduction to Jacobi Elliptic Functions	254
7.3*	Time-dependence of Pendulum in Terms of Elliptic Functions	257
7.4	Anharmonic Oscillations: Quartic Double-well Potential	259
7.5*	Quartic Oscillator: Exact Solution and Lindstedt-Poincaré Method	262
7.6	Problems	266
	References	269
8	Rigid Body Mechanics	271
8.1	Lab and Comoving Frames	273
8.2	Configuration Space and Degrees of Freedom	274
8.3	Infinitesimal Displacement and Angular Velocity of Rigid Body	275
8.4	Kinetic Energy and Inertia Tensor	277
8.5	Types of Rigid Bodies	280
8.6	Angular Momentum of a Rigid Body	282
8.7	Equations of Motion of a Rigid Body	283
8.8	Force-Free Motion of Rigid Bodies	286
8.9	Euler Angles and Rotations	291
8.10	Angular Velocity and Kinetic Energy in Terms of Euler Angles	293
8.11	Euler Equations for a Rigid Body in Body-Fixed Frame	296
8.12	Ellipsoid of Inertia and Qualitative Description of Free Motion of Rigid Body	301
8.13*	Solution of Force-Free Euler Equations Using Jacobi Elliptic Functions	305
8.14*	Poisson Bracket Formulation of Euler's Equations	308
8.15	Motion of a Heavy Symmetrical (Lagrange) Top	311
8.16	Problems	315
	References	320
9	Motion in Noninertial Frames of Reference	321
9.1	Uniformly Accelerating Frames and the Equivalence Principle	322
9.2	Nonuniformly Accelerated Frames: Lagrangian Approach	324
9.3	Uniform Rotation: Hamiltonian Formulation and Magnetic Analog	327
9.4	Precession of Foucault's Pendulum	328
9.5*	Circular Restricted Three-Body Problem	329
9.6	Problems	331
	References	332

10	Canonical Transformations	333
10.1	From Point Transformations to Canonical Transformations	335
10.2	Form of Hamilton's Equations is Preserved iff PBs are Preserved	337
10.3*	Brief Comparison of Classical and Quantum Mechanical Formalisms	341
10.4	Canonical Transformations: Area-Preserving Maps and Integral Invariants	344
10.5	CTs Preserve Poisson Tensor and Formula for PB of any Pair of Observables	347
10.6	Generating Function for Infinitesimal Canonical Transformations	349
10.7	Symmetries and Noether's Theorem in the Hamiltonian Framework	353
10.8	Liouville's Theorem	354
10.9*	Poincaré Recurrence	358
10.10	Generating Functions for Finite Canonical Transformations	363
10.11	Problems	370
	References	376
11	Angle-Action Variables	377
11.1	Angle-Action Variables for the Harmonic Oscillator	378
11.2	Generator of CT to Angle-Action Variables: Hamilton-Jacobi Equation	381
11.3	Generating Function for CT to Angle-Action Variables for Linear Oscillator	383
11.4	Angle-Action Variables for Systems with One Degree of Freedom	384
11.5*	Angle-Action Variables for Libration of the Simple Pendulum	385
11.6*	Bohr-Sommerfeld Quantization Rule	386
11.7*	Liouville Integrability and KAM Tori	388
11.8*	Liouville-Arnold Theorem	390
11.9*	Conserved Quantities from a Lax Pair	394
	11.9.1* Harmonic Oscillator Lax Pair	395
	11.9.2* Isospectral Evolution and Conserved Quantities	396
	11.9.3* Toda Chain: Flaschka's Variables and a Lax Pair	397
	11.9.4* Euler-Poinsot Top Lax Pair: Spectral Parameter	399
11.10	Problems	401
	References	403

12	Hamilton-Jacobi Equation	405
12.1	Time-Dependent Hamilton-Jacobi Evolution Equation	407
12.2*	Connection of Hamilton-Jacobi to Schrödinger and Eikonal Equations	411
12.3	Separation of Variables in the Hamilton-Jacobi Equation	412
12.4	Hamilton's Principal Function Is Action as a Function of Endpoint of a Trajectory	417
12.5*	Geometric Interpretation of HJ: Trajectories and HJ Wavefronts	418
12.6	Problems	421
	References	423
13	Normal Modes of Oscillation and Linear Stability	425
13.1	Elementary Examples of Coupled Small Oscillations	427
13.1.1	Normal Modes of Two Weakly Coupled Pendula Undergoing Small Oscillations	427
13.1.2	Normal Modes of a Diatomic Molecule	429
13.2	Double Pendulum: Formulation and Small Oscillations	430
13.2.1	Energy, Lagrangian and Equations of Motion	431
13.2.2	Normal Modes of a Double Pendulum	433
13.3*	Normal Modes Around a Static Equilibrium: General Framework	439
13.4	Small Perturbations Around a Periodic Solution	445
13.4.1	Formulation as a System of First Order Equations	447
13.4.2*	Time Evolution Matrix	448
13.4.3*	Monodromy Matrix	453
13.4.4*	Stability of a Periodic Solution	453
13.4.5*	Kapitza Pendulum with Oscillating Support: Mathieu Equation	455
13.5	Problems	458
	References	462
14	Bifurcations: Qualitative Changes in Dynamics	463
14.1	Bifurcations of Vector Fields on the Real Line	464
14.1.1*	Saddle-Node Bifurcation	464
14.1.2*	Transcritical Bifurcation	466
14.1.3*	Pitchfork Bifurcations	467
14.2	Bifurcations in Two Dimensions	472
14.2.1*	Saddle-Node, Transcritical and Pitchfork Bifurcations	472
14.2.2*	Hopf Bifurcations	475
14.3	Problems	478
	References	478

15	From Regular to Chaotic Motion	479
15.1	Chaos in Iterations of a Map	481
15.1.1	Lyapunov Exponent and Sensitivity to Initial Conditions	482
15.1.2	Chirikov-Taylor Standard Map: A Kicked Rotor	483
15.1.3	Logistic Map: Period Doubling, Chaos, Cantor Dust and Lyapunov Exponent	485
15.2*	Lyapunov Exponents for Continuous-Time Dynamical Systems	495
15.3*	Poincaré Return Map and Homoclinic Tangle	502
15.4	Hamiltonian Chaos: Order-Chaos-Order Transition in a Double Pendulum	503
15.4.1*	Poincaré Sections and Onset of Chaos	507
15.4.2*	Return to Regularity at High Energies	516
15.4.3*	Understanding the Zero Gravity Double Pendulum	517
15.5*	Chaos in Lorenz's Model for Convection	523
15.6	Problems	529
	References	533
16	Dynamics of Continuous Deformable Media	535
17	Vibrations of a Stretched String and the Wave Equation	537
17.1	Wave Equation for Transverse Vibrations of a Stretched String	539
17.2	Finite Differences: Wave Equation as a System of ODEs	543
17.3	Separation of Variables, Normal Modes and Solution by Fourier Series	544
17.4	Right- and Left-Moving Waves and d'Alembert's Solution	548
17.5	Conserved Energy of Small Oscillations of a Stretched String	551
17.6	Three Local Conservation Laws for the Wave Equation	553
17.7	Lagrangian and Hamiltonian for Stretched String	555
17.8	Conserved Quantities from Noether's Theorem	558
17.9	Dispersion Relation, Phase and Group Velocities	559
17.10*	Lax Pair for the First Order Wave Equation	561
17.11	Problems	564
	References	567
18	Heat Diffusion Equation and Brownian Motion	569
18.1	Obtaining the Heat Equation and Its Basic Properties	570
18.2	Solution of Initial Value Problem on an Interval by Fourier's Series	573
18.3*	Heat Kernel: Time Evolution Operator for Heat Equation	575
18.4	From Brownian Motion to the Diffusion Equation	580

18.4.1	Brownian Motion and the Atomic Hypothesis	580
18.4.2*	Random Walk Model and the Diffusion Equation	583
18.5	Problems	587
	References	588
19	Introduction to Fluid Mechanics	591
19.1	Fluid Element, Local Thermal Equilibrium and Dynamical Fields	592
19.2	Fluid Statics: Aero- or Hydrostatics	593
19.3	Flow Visualization: Streamlines, Streaklines and Pathlines	595
19.4	Material Derivative	597
19.5	Compressibility, Incompressibility and Divergence of Velocity Field	598
19.6	Local Conservation of Mass: Continuity Equation	601
19.7	Euler Equation for Inviscid Flow	602
19.8	Ideal Adiabatic Flow: Entropy Advection and Equation of State	606
19.9	Bernoulli's Equation	608
19.10	Sound Waves in Homentropic Flow	610
19.11	Vorticity and Its Evolution	612
19.12	Vortex Tubes: Kelvin and Helmholtz Theorems	615
19.13	Conservation of Energy, (Angular) Momentum and Helicity	617
19.14*	Hamiltonian and Poisson Brackets for Inviscid Flow	621
19.15*	Clebsch Variables and Lagrangian for Ideal Flow	626
19.16	Navier-Stokes Equation for Incompressible Viscous Flow	630
19.17	Problems	638
	References	646
Appendix A:	Mathematical and Kinematical Background	647
A.1	Vectors in Euclidean Space	648
A.2	Position Coordinates and Velocity and Acceleration Vectors	653
A.3	Circular Motion: Uniform and Nonuniform	654
A.4	Integration of Kinematical Equations: Uniform Acceleration	656
A.5	Plane Polar Coordinates	657
A.6	Spherical Polar Coordinates	660
A.7	Taylor Series	662
A.8	Some Vector Calculus: grad, div and curl	664
A.9	Stokes', Green's and Gauss' Integral Theorems	668
A.10	Vector Spaces, Matrices and Eigenvalue Problems	669
A.11*	Fourier Transform	676
A.12	Problems	678

Appendix B: Primer on Manifolds, Tensors and Groups	683
B.1 The Concept of a Manifold	683
B.2* Submanifolds, Connected and Simply Connected Manifolds	689
B.3* Smooth Functions or Scalar Fields	690
B.4* Vector Fields	692
B.5* Covector Fields or One-Forms	695
B.6* Tensors of Rank Two and Two-Forms	698
B.7* Higher Rank Tensor Fields and Forms	703
B.8* Pushforward and Pullback of Tensors	705
B.9* Exterior Algebra, Exterior Derivative and Bianchi's Identity	707
B.10* Integration on Manifolds and Stokes' Theorem	710
B.11* Covariant Derivative	714
B.12* Curvature on a Riemannian Manifold	716
B.12.1* Riemann-Christoffel Curvature Tensor	717
B.12.2* Geodesic Deviation and Riemannian Curvature	720
B.13* Groups, Lie Groups and Their Lie Algebras	722
B.14* Quaternions and the Axis-Angle Representation of Rotations	736
B.15 Problems	740
References	746
Supplementary Reading	747
References	749
Index	755

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