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Classical Mechanics

From Particles to Continua and Regularity to Chaos

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Preface

Why Classical Mechanics?

Classical[1] mechanics traces its origins to the study of the motion of bodies. It is an old subject whose basic principles were formulated in Galileo's and Newton's time (1600s). However, the subject remains alive and continues to thrive. This is because (a) it is the basis for the formulation of most physical systems and remains the predictive framework in which we try to understand and control the vast majority of physical phenomena despite the advent of the quantum theory,[2] (b) the theoretical edifice of mechanics has undergone repeated rejuvenation allowing it to be understood in increasingly deep ways (Lagrangians, Hamiltonians, Poisson brackets, symplectic forms, canonical transformations, Hamilton-Jacobi equation, perturbation theory, Dirac-Bergmann constraints, Lax pairs, Kolmogorov-Arnold-Moser (KAM) theory, symbolic dynamics, renormalization and so forth) and (c) old classical phenomena like turbulence are yet to be satisfactorily understood and new phenomena such as those surrounding integrable, chaotic, mixed and open systems pose fresh challenges. The history of classical mechanics also reveals that the theoretical and computational schemes developed to understand and solve Newton's equations of mechanics (such as conservation laws, variational principles, constraints, the Hamiltonian formulation or that in terms of geodesics or Hamilton-Jacobi wavefronts) have a utility beyond the validity of Newton's laws. They have even been used to discover and formulate other or more accurate laws of nature (such as those governing electromagnetism or the quantum dynamics of relativistic particles and fields) as well as to set up effective or statistical frameworks to deal with unexpected 'emergent' phenomena that arise in the presence of nonlinearities or when many degrees of freedom interact. Furthermore, developments in classical mechanics have been inspired by and used in numerous practical problems of science and engineering such as projectile motion, timekeeping,

[1] The adjective *classical* conveys that it is the oldest and best established branch of mechanics. Technically, the term *classical* is used to distinguish it from *quantum* mechanics.

[2] Even phenomena or systems that require a quantum mechanical treatment are often fruitfully first understood in some classical limit.

construction of roads, bridges, dams, musical instruments and gyroscopes, flight of airplanes, launching and placement of satellites and weather prediction. If we add to this the beauty of the mathematical form of classical dynamics, we begin to see why it remains a central part of physics research and education.

Scope of this Book

This book is meant primarily for postgraduate students of physics and their teachers. However, it should also be of use to undergraduates, researchers and others interested in mechanics, especially when viewed as a part of the larger fabric of theoretical physics. An attempt has been made to keep the book reasonably self-contained by developing topics from basic principles and by providing physical context, intuition and reasoning. Topics are presented in a manner that elucidates the thought process behind calculations. Mathematical constructions and terms are motivated and explained alongside physical developments. Readers should find them useful in thinking about physics and communicating effectively. It is hoped that these features will make the book suitable both for use in taught courses and self-study. Many details and subtle issues are clarified, which should enable teachers to explain ideas effectively. The process of learning mechanics is not linear: a novice is not expected to follow, at the first instance, every concept that is presented. Things tend to become clearer on subsequent readings. Footnotes are an integral part of this book and are used to avoid interrupting the flow. They contain qualifications, explanations, additional material, etc. While some of them are meant for instructors and advanced readers, many should be helpful even to first-time readers. Starred sections, on the other hand, contain topics that may be skipped on a first reading without affecting the continuity of the material. It is hoped that the reader will acquire a working knowledge of, affinity and appreciation for mechanics, its challenges and its connections to other areas of physics, as well as a degree of theoretical sophistication by working through this book.

Genesis of this Book

The specific suggestion to write this book for the *Texts and Readings in Physical Sciences* (TRiPS) series came from D. K. Jain of Hindustan Book Agency on 19 January, 2018, although he had mentioned it more casually earlier as well. He followed this up with periodic exhortations to start writing. The fact that both my guru S. G. Rajeev and my mama (maternal uncle) N. Mukunda (with E. C. G. Sudarshan) had written advanced books on mechanics [1] and [2] was a source of inspiration, but also made it a daunting task. Encouragement and feedback from students who took my courses, my colleagues H. S. Mani, K. P. N. Murthy, A. Laddha and K. G. Arun (who had seen my lecture notes) and D. Ghoshal helped convince me that it was all right to attempt yet another book on mechanics. Although my lecture notes had been with me for a few years, it was not until the day after new year's day, 15 April, 2020 that I started preparing the manuscript. The lock-down due to the COVID-19 pandemic played an important role in helping me get started and sustaining the effort. I soon realized how inadequate my rough lecture notes were, but it took longer for me to recognize just how much effort was needed to write this book! The process itself has been fun and rewarding and I hope readers will enjoy this book.

Acknowledgements

The classic text by Landau and Lifshitz [3] remains a good place to learn mechanics. I have occasionally relied on their exposition. My interest in mechanics and the viewpoint in this book owe a lot to what I learned from S. G. Rajeev. His work and writings continue to inspire me. I have also benefited from my collaboration with A. Thyagaraja, who has helped shape my viewpoint on fluid mechanics. Over the past decade, I have given lectures at several places in India (Chennai Mathematical Institute (CMI), Science Academy Refresher Courses and Workshops, etc.). The questions and comments from those who attended my lectures were useful in the preparation of my notes. I thank V. V. Sreedhar, G. Rajasekaran, H. S. Mani, M. Lakshmanan, C. Rao, A. N. Kumara, S. Ananth, K. Indulekha, N. V. Suryanarayana, D. Jatkar, M. Ashefas and others for asking me to deliver these lectures. Comments and suggestions from R. Jagannathan, R. Parthasarathy, G. Date, T. R. Ramadas, R. Nityananda, A. V. Khare, M. V. N. Murthy, J. K. Bhattacharjee, A. Das, A. Arul and anonymous reviewers have enriched the book, as have those of the Editors R. Ramaswamy and D. Ghoshal of TRiPS. Advice and suggestions from the publishers D. K. Jain and J. K. Jain of Hindustan Book Agency and S. Ahmad of Springer Nature have helped me prepare this book for publication. I am also grateful to my colleagues and the staff at CMI for the pleasant working environment.

Special thanks go to Ph.D. students Sachin Phatak, Sonakshi Sachdev, Himalaya Senapati, T. R. Vishnu, Ankit Yadav and Pritish Sinha as well as summer student Shivangi Dhiman with whom I have discussed many of the topics and exercises in this book. Shivangi helped prepare several of the figures and parts of the Index for the first three chapters. Her detailed study of the first drafts of these chapters helped iron out some obscurities. Ankit too helped with many figures, numerical calculations in Chaps. 8, 13 and 15 and gave much useful feedback on a majority of the chapters. Himalaya's critical comments on practically all chapters have significantly improved the exposition. He also helped with getting the LaTeX typesetting software package to do useful things! Sects. 2.8 and 9.5 on the three-body problem are based on the article [4] written with Himalaya. Pritish's questions upon reading portions of Chaps. 1, 2, 3, 4, 19 and Appendix B have helped clarify some subtle concepts. The introduction to quaternions in Appendix B.14 is based on the article [5] coauthored with Sonakshi. The discussion of Lax pairs in Chap. 11 and Chap. 17 is based on expository articles [6] and [7] written with Vishnu, who also made useful comments on Chap. 7.

I would like to acknowledge generous financial support in the form of a Ramanujan Fellowship, a Core Research Grant and a MATRICS grant from the Science and Engineering Research Board (SERB) of the Department of Science and Technology, Government of India, at various stages in the preparation of this book.

Finally, this book was made possible by the support I have received from my family, especially Krishnaswami, Leela and Maithreyi. It is written for Shraddha and dedicated to my parents and to Profs. C. S. Seshadri and G. Rajasekaran who unfortunately passed away while I was writing it.

Prerequisites

Although most mathematical methods are developed along the way, we assume some familiarity with calculus, vectors and matrices. Some of this background is in Appendix A while more may be found in [8, 9, 10, 11, 12, 13, 14, 15]. Connections to electromagnetism, optics, thermodynamics, relativity, quantum and statistical mechanics are occasionally pointed out. For those unfamiliar with these subjects, such remarks may simply be ignored. Finally, it is hoped that a reader can begin reading several of the chapters without having studied many of the preceding ones.

Arrangement of Material

The order of presentation is based on certain guiding principles. (1) Increasing number of degrees of freedom: point particles (1D–3D), rigid bodies and then continuous media. (2) A historical path through the development of the formalism of classical mechanics, progressing from the ideas of Galileo and Newton through the Bernoullis, d'Alembert, Maupertuis, Euler, Lagrange, Hamilton, Poisson, Jacobi, Lie, Hill, Poincaré, Lyapunov, etc. (3) Small oscillations to large oscillations: linear to nonlinear equations of motion. (4) The idea that the study of concrete examples often leads to general principles/formalisms which then inform on the behavior of other model systems. (5) Configuration and phase spaces that are topologically trivial (e.g., Euclidean spaces) followed by those that are not (e.g., circles, spheres, cylinders). (6) Increasing mathematical sophistication. (7) Regular dynamics to chaotic dynamics.

Outline of Chapters

For those who would like to begin with some mathematical and kinematical preliminaries, Appendix A deals with vectors in Euclidean space, kinematics, circular motion, polar coordinates, Taylor series, vector calculus, linear algebra and the Fourier transform. We begin our study of mechanics in Chap. 1 with the Newtonian formulation of the motion of a particle on a line and discuss some of the general phenomena that arise. The harmonic oscillator and pendulum are introduced as important examples of systems with one degree of freedom. The notions of oscillatory time period, time delay and inverse problems are introduced. Chapter 2 is devoted to Kepler's two-body central force problem which is used to motivate and illustrate the consequences of Newton's laws of mechanics and gravity in the context of planetary orbits. Replacing the gravitational force with the Coulomb force between charged particles, we derive Rutherford's differential cross section for the scattering of alpha particles by gold nuclei. The utility of Keplerian orbits goes beyond the two-body problem: we show how they turn up in the Euler and Lagrange solutions of the three-body problem! With this background at hand, we move (in Chap. 3) to the conceptual and formal development from Galileo's principle of relativity to the Newtonian, Euler-Lagrangian, Hamiltonian, Poisson bracket and Euler-Maupertuis formalisms of mechanics. Chapter 3 has been placed after Chaps. 1 and 2 in keeping with the adage that physics tends to be developed through the study of examples or model systems. General principles often emerge from the study of specific systems. A reader may of course go straight to Chap. 3 and return to the previous chapters when necessary. In Chap. 4, we point out the shortcomings of Newtonian mechanics in treating phenomena where speeds of particles or frames relative to an inertial frame are comparable to that of light and give a brief introduction to special relativistic mechanics. In Chap. 5, we return to nonrelativistic mechanics and adopt a dynamical systems viewpoint where we think of mechanical systems in terms of their associated vector fields on the state space. We then turn to a variety of examples in Chaps. 6, 7, 8, 9: small oscillations for one degree of freedom, (not necessarily small) oscillations of a simple pendulum and quartic anharmonic oscillator, rigid bodies and motion in noninertial frames (with applications to Foucault's pendulum and the restricted three-body problem). The formalisms developed in earlier chapters are put to use here, while new techniques (both exact and approximate) are also introduced. We then return to general structural aspects by discussing canonical transformations, angle-action variables and the Hamilton-Jacobi equation in Chaps. 10, 11, and 12, where we also meet Poincaré recurrence, Liouville integrability, KAM tori and Lax pairs. In Chap. 13, we go back to the study of small oscillations, this time around static and periodic solutions of systems with two or more degrees of freedom. Pendula coupled by a spring, a diatomic molecule and the double pendulum are used to illustrate normal modes, while the Kapitza pendulum provides a context to examine the stability of oscillations. In Chap. 14, we discuss several simple examples of bifurcations of vector fields, which illustrate how the qualitative behavior of a system can change as a control parameter is varied. This is followed in Chap. 15, by examples of discrete-time and continuous-time dynamical systems (standard and logistic maps, double pendulum and Lorenz oscillator) illustrating the passage from regular to chaotic motion. The next four chapters (Chaps. 16, 17, 18, 19) provide an introduction to continuum mechanics via d'Alembert's wave equation for vibrations of a stretched string, Fourier's equation for heat diffusion and Brownian motion and finally fluid mechanics. Appendix A discusses mathematical and kinematical preliminaries while Appendix B contains an informal introduction to concepts from the theory of manifolds, tensors, differential geometry, groups and Lie algebras that we use in our treatment of mechanical systems. We end with a list of books that treat related topics, cited references to the literature and a detailed index. Readers are encouraged to attempt the end-of-chapter problems, which are an integral part of the book. Each chapter begins with an introductory section providing conceptual context, outlining topics to be treated and mentioning further developments.

Planning a Course

A first course on mechanics for undergraduates has been taught by starting with background on vectors, kinematics, polar coordinates and vector calculus (Appendix A) followed by the framework of Newtonian mechanics (Sects. 3.1, 3.2), Galileo's relativity principle and Newton's laws (Sect. 3.3), concepts of phase space, conservation laws and their illustration via collisions (Sect. 3.4), motion in one dimension (Sects. 1.1, 1.2), Kepler's laws and Newton's law of gravity (Sect. 2.1), an introduction to the harmonic oscillator and simple pendulum (Sects. 1.1, 6.1, 1.5), uniformly accelerated frames (Sect. 9.1) and the elements of special relativity (Sects. 4.1, 4.2, 4.3, 4.4, 4.5, 4.6). A second course on mechanics could cover Lagrangian mechanics (Sects. 3.5, 3.6, 3.7, 3.8, 3.9, 3.10), Hamiltonian mechanics (Sects. 3.14, 3.15), Poisson brackets (Sect. 3.21), canonical transformations (Sects. 10.1, 10.4, time permitting), the gravitational two-body problem (Sects. 2.2, 2.3), small oscillations and normal modes (Sect. 13.1), damped harmonic oscillations (Sect. 6.4, time permitting), rigid bodies (Sects. 8.1, 8.2, 8.3, 8.4, 8.5) and noninertial frames of reference (Sect. 9.2). The content of a Master's course would depend on student background. It may be based on a selection from the following topics, with some being assigned as reading projects: motion in one dimension (Sects. 1.1, 1.2, 1.5), the Kepler problem (Sects. 2.1, 2.2, 2.3, 2.4, 2.5), the formalism of Newtonian, Lagrangian and Hamiltonian mechanics (Sects. 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.12, 3.14, 3.15, 3.18, 3.19, 3.21, 3.22), driven oscillations (Sect. 6.8) or anharmonic oscillations (Sect. 7.1), rigid body dynamics (Sect. 8.6, 8.7, 8.9, 8.10, 8.11, 8.12), canonical transformations (Sects. 10.1, 10.2, 10.4, 10.5, 10.6, 10.7, 10.8, 10.10), angle-action variables (Sects. 11.1, 11.2, 11.3, 11.4) and the Hamilton-Jacobi equation (Sects. 12.1, 12.3, 12.4). A course on nonlinear dynamics could treat vector fields in one and two dimensions (Sects. 5.1, 5.2, 5.3, 5.4), planar vector fields arising from small oscillations (Sects. 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7), the pendulum (Sect. 7.1) or anharmonic oscillator (Sects. 7.4, 7.5), infinitesimal canonical transformations (Sect. 10.6), Liouville's theorem (Sect. 10.8), Poincaré recurrence (Sect. 10.9), angle-action variables (Sects. 11.1, 11.2, 11.3, 11.4), Liouville integrability and KAM tori (Sect. 11.7), normal modes (Sect. 13.2), stability of periodic solutions (Sect. 13.4), bifurcations of vector fields (Sects. 14.1, 14.2) and chaos (Sects. 15.1, 15.4). An introductory continuum mechanics course (sans elasticity) may be based on the unstarred sections in Chaps. 16, 17, 18, 19. Mathematical supplements from Appendix B may be introduced when the need arises or in a course on mathematical methods of physics. In planning courses, instructors should feel free to omit passages based on student background and time available. Many chapters are self-contained, so teachers may treat topics in an order they are comfortable with. For instance, motion in noninertial frames could be discussed before rigid bodies, although the corotating frame of a rigid body is a nice example of a noninertial frame. Finally, I hope to maintain a list of corrections/clarifications and additional resources at the web address: http://www.cmi.ac.in/~govind.

Chennai, India
May 2024

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About This Book

This well-rounded and self-contained treatment of classical mechanics strikes a balance between examples, concepts, phenomena and formalism. While addressed to graduate students and their teachers, the minimal prerequisites and ground covered should make it useful also to undergraduates and researchers. Starting with conceptual context, physical principles guide the development. Chapters are modular and the presentation is precise yet accessible, with numerous remarks, footnotes and problems enriching the learning experience. Essentials such as Galilean and Newtonian mechanics, the Kepler problem, Lagrangian and Hamiltonian mechanics, oscillations, rigid bodies and motion in noninertial frames lead up to discussions of canonical transformations, angle-action variables, Hamilton-Jacobi and linear stability theory. Bifurcations, nonlinear and chaotic dynamics as well as the wave, heat and fluid equations receive substantial coverage. Techniques from linear algebra, differential equations, manifolds, vector and tensor calculus, groups, Lie and Poisson algebras and symplectic and Riemannian geometry are gently introduced. A dynamical systems viewpoint pervades the presentation. A salient feature is that classical mechanics is viewed as part of the wider fabric of physics with connections to quantum, thermal, electromagnetic, optical and relativistic physics highlighted. Thus, this book will also be useful in allied areas and serve as a stepping stone for embarking on research.

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Govind S. Krishnaswami is a Professor of Physics at the Chennai Mathematical Institute, India, and is a recipient of the Apker Award and Marie Curie, EPSRC and Ramanujan Fellowships. His research in areas such as particle physics, nonlinear dynamics and fluid mechanics has often exploited ideas and tools from classical mechanics. He has taught a wide variety of subjects and is known for the clarity of his lectures and pedagogical articles in Resonance, a journal of science education.