Classical Mechanics: From Particles to Continua and Regularity to Chaos

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Book website http://www.cmi.ac.in/~govind/cm-book

List of clarifications, changes and corrections

1. page xiv. Added sentence to Preface: Finally, I hope to maintain a list of corrections/clarifications and additional resources at the web address:

   http://www.cmi.ac.in/∼govind.

2. p26 Footnote 12: Correction: "Kepler's 2nd law provides information on the latter." Was mistakenly mentioned as 3rd law.

3. page 52. Problem 2.9, Eqn (2.82). Correction of typo in formula for relative momentum "$\boldsymbol{p} = \mu_1 \boldsymbol{p}_2 - \mu_2 \boldsymbol{p}_1$". Was wrongly given as $\boldsymbol{p} = \boldsymbol{p}_2/\mu_2 - \boldsymbol{p}_1/\mu_1$.

4. page 57. Sect. 3.3, 3rd line: added missing word *is*: 'This is not an assumption but rather . . . '

5. p83 end of Sect. 3.10. Added sentence and reference: "See also the discussion and examples in [89]." Mallesh K S et. al., Resonance **16**(2), 129 (2011).

6. p103, Sec 3.19, below (3.119): added sentence "Conditions for extrema are the same even if $H = H(q, p, t)$." In other words, Hamilton's equations apply even if H is explicitly time-dependent.

7. p140 Prob 3.25(b). Removed the erroneous symbol $\langle \mathbf{4} \rangle$.

8. p175 above (5.7) changed ODES → ODEs.

9. p175 revised sentence below (5.8): "A change of coordinates can convert a linear vector field into a nonlinear one (though the opposite cannot always be done)." Added new Footnote 5: The desired linearizing transformation may not exist or may not be nice enough (e.g., smooth). For instance, if one tries to construct it using a series, the series may not converge.

10. p179 below (5.17), added phrase "of strength $\gamma = 1$" to sentence: "We may view . . . as an inertialess limit . . . subject to a damping force of strength $\gamma = 1$."

11. p182 Sect. 5.3, above (5.20): added parenthetical remark: "Indeed, integrating (assuming $x \not\equiv 0$), we find $2\sqrt{x} = t + \text{constant}$." If $x(t)$ is identically zero for some time, we cannot divide by it while separating variables to perform the integral.

12. p190 Sect. 5.4*: replaced positive with nonnegative in definition of Lyapunov function: (a) L is nonnegative on the phase space.

13. p201 in (6.10): removed boldface $\boldsymbol{r}_+ \to r_+$.

14. p205 Sect. 6.5 below (6.25): corrected formula: "slope of trajectories is given by $dy/dx = \dot{y}(t)/\dot{x}(t) = y(0)/(x(0) + y(0)(t + 1/\lambda))$."

15. p210 Sect. 6.7*: added definition of elliptic fixed point: "... a *hyperbolic fixed point* is one at which all the eigenvalues of the linearization have nonzero real parts (this applies to real autonomous systems of any dimension). A fixed point is *elliptic* if the eigenvalues are imaginary."

16. p232: above (7.23): typo: "... sn must be odd and ...". Word 'be' was missing.

17. p239: (7.55) $x_1(t) = (a^3/\omega_0^2)((1/32)\cdots)$. $1/\omega_0^2$ was missing in the prefactor.

18. p318: Typographical error in (10.48). Equations should read:

$$Q^i \equiv q^i(t+\delta t) \approx q^i(t) + \delta t \frac{\partial H}{\partial p_i} \quad \text{and} \quad P_i \equiv p_i(t+\delta t) \approx p_i(t) - \delta t \frac{\partial H}{\partial q^i}. \quad (1)$$

19. p394: added Footnote 13 above (13.41) concerning the equation $H = H_+ + H_-$. "Although we could check this by direct calculation, we will use matrix methods that also apply to more degrees of freedom and reveal certain useful linear algebraic structures."

20. p405 Eqn (13.80). Corrected symbol for identity matrix from 1 to I

21. p405 Eqn (13.81) and sentence above it revised: To get ψ after a longer time, we compose time evolution over several short times $t/n = \epsilon$:

$$\psi(t) = \lim_{n\to\infty} (I+\epsilon A((n-1)t/n))(I+\epsilon A((n-2)t/n)) \dots (I+\epsilon A(t/n))(I+\epsilon A(0))\psi(0). \quad (2)$$

    The factors of epsilon were missing and identity I was denoted 1.

22. p433 para above Sect. 15.1.1: replaced "open subsets of" with "2d regions in" to clarify notion of chaotic orbits: "wander around a bounded portion of M in an irregular fashion and can seem to fill up 2d regions in M".

23. p438 Sect. 15.1.2: added parenthetical remark defining elliptic fixed point: "... the elliptic fixed point (Jacobian characteristic polynomial discriminant $<$ 0) at the origin."

24. p441 para above (15.12): changed $\lesssim$ to $<$ in 2nd inequality: "... one finds that for $r_2 \approx 3.449 \lesssim r < r_3 \approx 3.544$ there is a stable ...".

25. p443 Fig 15.7 caption: changed $=$ to $\approx$: "onset of chaos at $r_\infty \approx 3.569946$."

26. p479 Prob. 15.10 rewrote $(\pm\pi, 0)$ as $(\pi, 0)$ so that the fixed point lies within the 'closed-open' fundamental domain $[0, 2\pi) \times [0, 2\pi)$.

27. p483 Chapt 16 first sentence. Prefixed "The subject of" to sentence. "The subject of continuum mechanics begins by dealing with ...".

28. p532 Sect. 19.1 1st para. removed qualifier 'essentially' for infinite: "Thus, we model a fluid as a continuum system with an infinite number . . . ".

29. p533 first sentence of Sect. 19.2: removed the qualifier 'inertial': ". . . when the fluid is not in motion in the frame considered."

30. p534 Sect. 19.2 corrected typo 'heats' in last sentence of 1st para: " . . . is the ratio of heat capacities . . . ".

31. p534 last para, **Example: Free surface of a rotating liquid.** Added an initial phrase to the 3rd sentence: "In a corotating frame, the body forces . . . ".

32. p535 first para 1st sentence of Sect. 19.3: Removed "(which can be rather jagged, as observed in Brownian motion)". Brownian motion reveals jagged trajectories of pollen grains, rather than of fluid molecules.

33. p536 Fig 19.3c: Changed figure of streakline to avoid self-intersection.

34. p536 Sect. 19.3: Added at the end of **Streaklines** para: "A streakline cannot self-intersect."

35. p536 Footnote 7: added word 'indirectly': ". . . used by Robert Brown (1827) to indirectly reveal the random thermal motion . . . ". One directly sees the motion of pollen grains, not water molecules.

36. p539 Footnote 12. Clarified definition of compressibility. Replaced "To make compressibility independent of the size of the fluid element, we divide by its volume to arrive at an intensive variable κ." with "To obtain a nontrivial limit as $V \to 0$, we divide by the volume V of the fluid element to arrive at the local (intensive) variable κ."

37. p544 Sect. 19.8, 7^{th} line: should be molecular mass m, not μ in parenthetical remark: "($p = \rho k_b T/m$ for an ideal gas with molecular mass m.)"

38. p548 Eqn. (19.41): reordered the equations for small variations to match the text: velocity followed by pressure and density.

39. p549 Eqn. (19.47): subscript 's' was missing from speed of sound c_s.

40. p550 Sect. 19.11, 1^{st} para 7^{th} line. Added Cartesian formula: "velocity field $\boldsymbol{v}(r, \theta, z) = \boldsymbol{\Omega} \times \boldsymbol{r} = \Omega(x\hat{y} - y\hat{x}) = \Omega r\hat{\theta}$ in cylindrical coordinates"

41. p550 Sect. 19.11, 1^{st} para 10^{th} line. Clarified origin of Stokes' factor of 2: "$\boldsymbol{w} = \Omega(\partial_x x + \partial_y y)\hat{z} = 2\Omega\hat{z}$ has a magnitude of twice the angular speed Ω."

42. p553 Eqn. (19.60) typographical error corrected. Line integral of gradient, not divergence: "$\frac{d\Gamma}{dt} = \oint_{C_t} \boldsymbol{\nabla}\left(-h - \varphi + \frac{1}{2}\boldsymbol{v}^2\right) \cdot d\boldsymbol{r} = 0$".

43. p555 Footnote 37 penultimate sentence. Revised: "What is more, from equipartition [120], $U = (f/2)Nk_bT$ so $C_v = (f/2)Nk_b$ where f is the number of degrees of freedom ...". Correction: "$(5/2)Nk_b$ for diatomic" (not 7/2). Clarification added at end of sentence: "(without vibrations)".

44. p556 2nd sentence below (19.66). Removed 'or' between homentropic and barotropic. "This formula holds for adiabatic flow as well as the special case of homentropic barotropic flow where p is a function of ρ."

45. p559 Barotropic flow PBs, below (19.77). Added comments on antisymmetry, Leibniz rule, Jacobi identity and dimensional analysis. "These noncanonical PBs are extended by postulating linearity and the Leibniz rule (see Phys. Plasmas **23**, 022308 (2016) for the Jacobi identity). Being associated with positions and momenta of distinct particles, variables at distinct locations commute. At the same location, the PB can diverge due to the Dirac δ function (see Appendix A.11*). While the $\{v_i, v_j\}$ PB being proportional to $\epsilon_{ijk}w_k/\rho$ ensures antisymmetry and the correct dimensions, that between $\boldsymbol{v}$ and ρ is a 'constant' vector, the gradient of $\delta(\boldsymbol{x} - \boldsymbol{y})$ is not dependent on dynamical variables."

46. p565 below (19.112). Correction: "Taking its gradient ...", (not divergence).

47. p567 Footnote 54: added word 'fixed': "...is the outward normal on fixed boundaries."

48. p568 first line: clarified interpretation of deviatoric stress tensor. "The non-isotropic deviatoric stress tensor $d_{ij} = p\delta_{ij} - T_{ij}$ can make an element change shape but not volume. "

49. p568 Footnote 60: Added the word 'Newtonian': "This is the Newtonian fluid analog of Hooke's law for an elastic solid: ..."

50. p568 Footnote 62: replaced last two sentences with: "In an isotropic fluid, the viscosity tensor A_{ijkl} must be an isotropic tensor. Its components must be the same in all Cartesian frames."

51. p570 below (19.125): revised parenthetical remark: "(e.g., in slow creeping flow relevant to swimming microbes [104])" and added reference E M Purcell.

52. p571 1st para line 10 corrected: "In other words, the drag force is proportional to speed at low speeds." It is the drag force rather than the drag coefficient.

53. p574 Prob. 19.3 added the qualifier 'mean' since air is a mixture of oxygen and nitrogen: "...where μ is the mean molar mass."

54. p580 Added new problem 19.50: "Suppose we have the **no-slip boundary condition** on the velocity field $\boldsymbol{v}$ on a fixed solid plane boundary $z = 0$ for flow in the half space $z \geq 0$. What can you say about the vorticity on the boundary?"

55. p580 Added new problem 19.51: "Suppose we rotate a Cartesian frame for $\mathbb{R}^3$ to a new frame via an orthogonal transformation L. Given a **Cartesian tensor** with components $t_{ij\ldots k}$, its components in the new frame are $\tilde{t}_{i'j'\ldots k'} = L_{i'i}L_{j'j}\cdots L_{k'k}t_{ij\ldots k}$ with repeated indices summed from one to three. (a) Express the condition for L to be orthogonal in components. (b) What is $\det L$? (c) Show that the Kronecker symbol has the same components in the new frame: $\tilde{\delta}_{i'j'} = \delta_{i'j'}$ so that it is an **isotropic tensor**. (d) Show that the Levi-Civita symbol ϵ_{ijk} is also isotropic. Hint: use $\epsilon_{ijk}L_{1i}L_{2j}L_{3k} = \det L$."

56. p580 Added new problem 19.52: "**Buoyancy.** Suppose a body of volume V_b is fully submerged in a fluid of constant density ρ at rest and subject to Earth's downward acceleration due to gravity $-g\hat{z}$ where $\hat{z}$ points upwards. Find the force due to fluid pressure on the body."

57. p593 Sect. A.6: added missing square-root: "This projected vector has length $r\sin\theta = \sqrt{x^2+y^2}$."

58. p602 above (A.48) added summation symbol in the formula $L(v) = \sum_j v_j L(e_j)$.

59. p603: above (A.51) added parenthetical remark: "(henceforth, repeated indices are summed over their range)".

60. p606. Added examples at end of para before Cayley-Hamilton theorem. "When eigenvalues coincide as a parameter is varied, the corresponding eigenvectors may remain independent (as in $(1+\delta, 0|0, 1-\delta)$) or become collinear (e.g., $(1+\delta, 1|0, 1+2\delta)$)."

61. p607. 4th line revised to define the commutator: (iv) L is normal, i.e., it commutes with its transpose ($[L, L^t] = LL^t - L^tL = 0$).

62. p607. Eq. (A.61) rearranged:

$$\begin{aligned} LS &= S\Lambda \quad \text{where} \quad S = \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix}, \\ LS &= \begin{pmatrix} Lv_1 & Lv_2 & \cdots & Lv_n \end{pmatrix} \quad \text{and} \quad S\Lambda = \begin{pmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \cdots & \lambda_n v_n \end{pmatrix}. \end{aligned} \tag{3}$$

63. p609. Prob. A8 revised: "Show that vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ point in different directions (not parallel/antiparallel) if and only if $a\boldsymbol{u} + b\boldsymbol{v} = \boldsymbol{0}$ implies that both $a, b = 0$."

64. p609. Prob. A9 split into (a) and (b): "(a) Show that three vectors in $\mathbb{R}^2$ cannot be linearly independent. (b) In $\mathbb{R}^n$ what is the maximum number of vectors that can be linearly independent?"

65. p610. Prob. A.10 revised: Express the Cartesian components of the cross product $\boldsymbol{a} \times \boldsymbol{b}$ in terms of those of $\boldsymbol{a}$ and $\boldsymbol{b}$ by using $\hat{x} \times \hat{y} = \hat{z}$, etc.

66. p610. Prob. A13 reworded: **Scalar triple product and Levi-Civita symbol.** If a_i, b_j, c_k are the Cartesian components of three vectors in $\mathbb{R}^3$, then verify that $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \sum_1^3 \epsilon_{ijk} a_i b_j c_k$.

67. p610. Prob. A14 made a starred problem.

68. p610. Prob. A16 split into (a) and (b). "(a) Suppose Newton ... What are the coordinates of the particle according to Newton? (b) What are ... his coordinate axes were rotated counterclockwise by $\pi/4$ relative to those of Galileo?"

69. p611. Prob. A.22(b). Quadratic Taylor coefficient should be $3/8$, not $1/8$.

70. p612. Prob. A.34. Added series for matrix exponential: **Exponential.** Suppose A is diagonalizable via a similarity transformation. Use this to find an expression for $e^A = I + A + A^2/2! + \cdots$ and apply it to $A = i\theta\sigma_1$.

71. p613. Footnote 1: Reworded 2nd sentence. "Points on the boundary will not have open neighborhoods $\cong \mathbb{R}^n$ and need to be treated differently."

72. p613. Footnote 2 reworded. "The open neighborhoods we have in mind are simple ones: they must come in one piece and be contractible (shrunk continuously to a point). Examples: In 1d they are continuously deformed (stretched/bent) versions of open intervals (a, b) on the real line. In 2d and 3d they are continuous deformations of the open disk $x^2 + y^2 < 1$ and open ball $x^2 + y^2 + z^2 < 1$. The open interval, disk and ball is each continuously deformable into $\mathbb{R}$, $\mathbb{R}^2$ and $\mathbb{R}^3$. Similarly, we have open balls in higher dimensions. Our neighborhoods look like them. By contrast, ..."

73. p614. Reworded caption of Fig. B.1(b): "(b) The closed interval $[0, 1]$ is *not* a manifold as 0 and 1 do not have open neighborhoods that look like $\mathbb{R}$: it is a manifold with boundary.

74. p615. Footnote 5 reworded. "'Closed-open' neighborhoods of 0 such as $[0, 1/2)$ are not homeomorphic to $\mathbb{R}$."

75. p618. Footnote 11: corrected 2nd sentence. "The row (column) rank of a matrix is the number of linearly independent rows (columns). They can be shown to be equal and this common number is called the rank of the matrix."

76. p639. 4 lines above (B.52): typo: "... where $J^i_j = \frac{\partial y^i}{\partial x^j}$ is the Jacobian matrix." (was wrongly mentioned as inverse Jacobian).

77. p649. Appendix B.13* in **Definition**: changed "the strong nuclear force ... is determined by a gauge symmetry principle" $\rightarrow$ "the strong nuclear force among quarks and gluons is determined by a 'gauge principle' ". Gauge transformations are not an ordinary symmetry but a redundancy in the description.

78. p658. Penultimate sentence of **Circle group** $U(1)$**.** paragraph. Added qualifier 'finite': "Since $U(1)$ is abelian, all its subgroups are abelian, the finite ones are the cyclic groups C_n for $n = 1, 2, \cdots$."

79. p661. Word before (B.102). Changed 'is' to 'are'. "... Lie brackets among the basis elements in (B.100) are".

80. p665. Problem B2: Above (B.114) changed "punctured plane $[(x, y) \neq (0, 0)]$" to "plane with negative x-axis ($x \leq 0, y = 0$) removed". Below (B.114) removed the phrase "on the punctured plane".

81. p676. Added reference [89]. "Mallesh K S, Chaturvedi S, Balakrishnan V, Simon R and Mukunda N, *Symmetries and Conservation Laws in Classical and Quantum Mechanics, 1. Classical Mechanics*, Resonance **16**(2), 129 (2011)." Referred to on page 83.

82. p677. Added reference [104]. "Purcell E M, *Life at low Reynolds number*, Am. J. Phys. **45**, 3 (1977)." Referred to on page 570.