Introduction to Kac-Moody Lie Algebras

- **Instructor:** Debajyoti Nandi
- **Email:** djn (at) cmi (dot) ac (dot) in
- **Office:** NB-306

Description

The objective of this introductory course is to develop a general theory of Kac-Moody Lie algebras leading up to the Weyl-Macdonald-Kac formulas. There are interesting connections (in ideas, impetus and motivation) between the theory of Kac-Moody Lie algebras and algebraic combinatorics (especially, partition identities), geometry, number theory, as well as theoretical physics (e.g., string theory and conformal field theory). My interest (and bias) in this subject stems from the connection to partition identities (of integer partitions), exemplified by the vertex-operator-theoretic interpretation and proof (Lepowsky, Milne, Wilson) of the classical Rogers-Ramanujan identities (as well as many recent developments).

Prerequisite

This course will be logically self-contained. Basic familiarity with linear algebra and abstract algebra will be assumed, but no prior knowledge of Lie algebras will be assumed. Familiarity with finite dimensional (semisimple) Lie algebras and their representations may be *useful but not required*. In fact, both can be learnt simultaneously with a bit of mathematical maturity and curiosity.

Syllabus

- Definitions
- The Weyl groups
- Standard modules
- The numerator formula
- Symmetrizability and the invariant form
- The Casimir operator and consequences
- The Weyl-Macdonald-Kac formulas

Assessment

- Final: 40%
- Midterm: 35%
- Assignments: 25%

Textbook

There is no prescribed textbook for this course. This course will be based on Lepowsky's Paris notes (unfortunately, not available publicly). Kac's book is a standard textbook on this subject. For semisimple Lie algebras and their representations see Humphreys and Fulton-Harris. For additional reading suggestion see the references section below. There are also many lecture notes available online.

References

Andrews, "Theory of Partitions"

Di Francesco, Mathieu and Senechal, "Conformal Field Theory"

Fulton and Harris, "Representation Theory"

Humphreys, "Introduction to Lie Algebras and Representation Theory"

Kac, "Infinite Dimensional Lie Algebras"

Lepowsky, "Introduction to Kac-Moody Lie Algebras" (Paris Notes)