Berlin · New York 2005

Krull-Schmidt reduction for principal bundles

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Abstract. We prove a principal bundle analog of a theorem on vector bundles that says that a vector bundle on a projective variety is isomorphic to a unique direct sum of indecomposable vector bundles (unique up to a permutation of the direct summands).

1. Introduction

An algebraic vector bundle V is called indecomposable if it is not isomorphic to a direct sum of vector bundles of positive rank. A well known theorem of Atiyah states that any vector bundle V over an irreducible projective variety can be expressed as a direct sum of indecomposable vector bundles, and furthermore, in any two such expressions of V, the same indecomposable components appear the same number of times [At1].

Let G be a linear algebraic group over an algebraically closed field k of characteristic zero. In Section 2 we define a notion of L-indecomposability (here L stands for Levi) of a principal G-bundle over an irreducible projective variety over k. For G = GL(n,k), the notion of L-indecomposability of a G-bundle coincides with the notion of indecomposability of the associated rank n vector bundle.

Let G be a reductive group. By a Levi subgroup of G we will mean a reductive subgroup H of some parabolic subgroup P of G such that H projects isomorphically onto P quotiented by its unipotent radical $R_u(P)$.

We prove that any principal G-bundle E admits a reduction of structure group $E_H \subset E$ to a Levi subgroup H of G with the property that E_H is an L-indecomposable H-bundle (see Theorem 3.2). Furthermore, if E_{H_1} is another such reduction, then H is isomorphic to H_1 by an inner automorphism of G and the principal G-bundle G

If $k = \mathbb{C}$ and the base is a compact connected complex manifold, then the above results remain valid (Remark 3.5).

The method of proof in [At1] is special to vector bundles. The method used here gives a new proof even in the case of vector bundles.

We thank Kapil H. Paranjape for some helpful discussions.

2. The L-indecomposable principal bundles

Let k be an algebraically closed field of characteristic zero. Unless mentioned otherwise all the objects that we consider are over k.

Let G be a connected reductive linear algebraic group over k. A closed algebraic subgroup $P \subset G$ will be called a *parabolic subgroup* if G/P is complete. So G itself is considered as a parabolic subgroup of G.

For a parabolic subgroup P, its unipotent radical will be denoted by $R_u(P)$. So, $P/R_u(P)$ is reductive. There is a reductive subgroup L(P) of P that projects isomorphically to $P/R_u(P)$. In fact, if we fix a maximal torus T of G contained in P, then L(P) can be taken to be the maximal reductive subgroup of P invariant under the adjoint action of P on P. Any two reductive algebraic subgroups of P that project isomorphically to $P/R_u(P)$ are conjugate in P. If P = G, then L(P) = G.

By a *Levi subgroup* of G we will mean a reductive algebraic subgroup of some parabolic subgroup P of G that projects isomorphically to $P/R_u(P)$.

Let M be an irreducible projective variety over k. Let H be a linear algebraic group over k. A *principal H-bundle* over M is an algebraic variety E equipped with a surjective affine morphism $p: E \to M$ satisfying the following conditions:

- (1) the morphism p is flat;
- (2) the variety E is equipped with a right action $\psi : E \times H \to E$ of H such that $p \circ \psi = p \circ \pi_1$, where π_1 denotes the natural projection of $E \times H$ to E;
 - (3) the map

$$(\pi_1, \psi): E \times H \to E \times_M E$$

to the fiber product is an isomorphism.

Given an *H*-bundle *E* and an algebraic subgroup $H_0 \subset H$, a reduction of structure group of *E* to H_0 is a section of the projection $E/H_0 \to M$.

Given such a section

$$\sigma: M \to E/H_0$$
.

consider the inverse image $q^{-1}(\sigma(M)) \subset E$, where $q: E \to E/H_0$ is the quotient map. It

is easy to see that $q^{-1}(\sigma(M))$ is preserved by the action of H_0 on E, and furthermore, $q^{-1}(\sigma(M))$ is a principal H_0 -bundle over M.

For a G-bundle E over M, where G is a connected linear algebraic group over k, consider the quotient

$$Ad(E) := \frac{E \times G}{G}$$

for the diagonal action of G, with G acting on itself (from the right) by inner conjugation. The conjugation action (from the right) of any $g_0 \in G$ on G is defined by $g \mapsto g_0^{-1}gg_0$.

Note that Ad(E) is a group scheme over M with fibers isomorphic to G. More precisely, the fibers of Ad(E) are identified with G up to an inner conjugation. Let Aut(E) denote the space of all global sections of the adjoint bundle Ad(E). Therefore, Aut(E) gets the structure of a group. Since M is a complete variety, Aut(E) is an algebraic group over k. In fact, Aut(E) is the group of all G bundle automorphisms of E over M [Gr], p. 82.

Any $\tau \in \operatorname{Aut}(E)$ gives an automorphism of E that commutes with the action of G. Indeed, for any point $x \in M$, if $(z,g) \in p^{-1}(x) \times G \subset E \times G$ represents $\tau(x)$, then the automorphism of E defined by τ sends z to zg.

Let

$$(2.1) Aut^0(E) \subset Aut(E)$$

denote the connected component of Aut(E) containing the identity element. Let

$$(2.2) T(E) \subset \operatorname{Aut}^0(E)$$

be a maximal torus of $\operatorname{Aut}^0(E)$. Any two maximal tori of $\operatorname{Aut}^0(E)$ are conjugate in $\operatorname{Aut}^0(E)$ [Bo], p. 148, Corollary 11.3(1). In particular, they are isomorphic.

Let Z(G) denote the center of G. The connected component of Z(G) containing the identity element will be denoted by $Z_0(G)$.

Definition 2.1. A *G*-bundle *E* over *M* is called L-*indecomposable* if a maximal torus T(E) of $Aut^0(E)$ is isomorphic to $Z_0(G)$, the connected component of the center of *G*. If a *G*-bundle *E* over *M* is not L-indecomposable then we say that *E* is L-*decomposable*.

Remark 2.2. Note that the center Z(G) is contained in $\operatorname{Aut}(E)$. Indeed, as action of any $g \in Z(G)$ on E commutes with the action of G, it defines an automorphism of E. Therefore, $Z_0(G)$ is contained in $\operatorname{Aut}^0(E)$. It is easy to see that $Z_0(G)$ is contained in the center of $\operatorname{Aut}^0(E)$, and hence it is contained in any maximal torus of $\operatorname{Aut}^0(E)$. Therefore, if $Z_0(G)$ is isomorphic to a maximal torus T(E) of $\operatorname{Aut}^0(E)$, then it is a maximal torus of $\operatorname{Aut}^0(E)$ by the inclusion map.

Remark 2.3. If G is a torus then the above definition implies that any G-bundle is L-indecomposable.

Proposition 2.4. A principal G-bundle E over M is L-indecomposable if and only if the quotient $\operatorname{Aut}^0(E)/Z_0(G)$ is a unipotent group.

Let G be a connected reductive linear algebraic group. A principal G-bundle E over M is L-indecomposable if and only if E does not admit a reduction of structure group to a proper Levi subgroup of G.

Proof. If a maximal torus of $\operatorname{Aut}^0(E)$ is isomorphic to $Z_0(G)$, as $Z_0(G)$ is in the center of $\operatorname{Aut}^0(E)$, the maximal torus of the quotient $\operatorname{Aut}^0(E)/Z_0(G)$ is the trivial group. In other words, $\operatorname{Aut}^0(E)/Z_0(G)$ is a unipotent group. In the converse direction, since $Z_0(G)$ is contained in any maximal torus of $\operatorname{Aut}^0(E)$, if $\operatorname{Aut}^0(E)/Z_0(G)$ is unipotent, its maximal torus being trivial, $Z_0(G)$ coincides with a maximal torus of $\operatorname{Aut}^0(E)$. This proves the first part of the proposition.

To prove the second part of the proposition assume that E admits a reduction of structure group to a proper Levi subgroup $H \subseteq G$. Fix such a reduction $E_H \subset E$. Note that $\operatorname{Aut}^0(E_H)$ is a subgroup of $\operatorname{Aut}^0(E)$. Indeed, since E_H is a reduction of structure group of E, we have $E \cong E_H \times^H G$. Therefore, an automorphism of E_H extends to an automorphism of E by using the identity automorphism of E.

If H projects isomorphically onto a proper Levi factor of a parabolic subgroup of G, then $Z_0(H)$, namely the connected component of the center of H containing the identity element, is strictly larger than $Z_0(G)$. Since $Z_0(H) \subset \operatorname{Aut}^0(E_H) \subset \operatorname{Aut}^0(E)$, this implies that E is L-decomposable.

In the converse direction, assume that E is L-decomposable. This implies that G is nonabelian.

Take any automorphism $\psi \in T(E)$, where T(E) as in (2.2) is a maximal torus of $\operatorname{Aut}^0(E)$. For any point $x \in M$, the evaluation map

$$e_x : \operatorname{Aut}^0(E) \to \operatorname{Ad}(E)_x \cong G$$

is a morphism of algebraic groups. Consequently, $e_x(\psi)$ is a semisimple element of G (see [St], p. 32, Proposition 3).

Now, the space of all conjugacy classes of semisimple elements in G is parametrized by the quotient T/W(T), where $T \subset G$ is a maximal torus and W(T) is the corresponding Weyl group. Note that T/W(T) is an affine variety. Since the variety M is complete and irreducible, the conjugacy class of $e_x(\psi)$ is independent of x. Note that the conjugacy classes of $Ad(E)_x$ are naturally in a bijective correspondence with the conjugacy classes of G.

Consider the collection of all elements g in a given maximal torus of G with the property that the centralizer of g is not a proper Levi subgroup of G. This collection, which we will denote by \mathcal{B} , is closed under multiplication by Z(G). It is known that the quotient $\mathcal{B}/Z(G)$ is a finite set (see [DM], p. 113).

Observe that the image $e_x(T(E))$ contains the center Z(G) properly provided we have

dim $T(E) > \dim Z(G)$. This is because if the image $e_x(T(E)) = Z(G)$ (using $Ad(E)_x \cong G$), then from the fact that the conjugacy class of the image is independent of the point x it follows that we have T(E) = Z(G) as a subgroup of $Aut^0(E)$.

In view of the above observations and the given condition dim $T(E) > \dim Z(G)$ (as E is L-decomposable), we can choose $\psi \in T(E)$ in such a way that the centralizer of $e_x(\psi)$ is a *proper Levi subgroup* of $\mathrm{Ad}(E)_x \cong G$. (This fact can also be deduced from the proof of [St], p. 98, Proposition 4.) So we will assume that the centralizer of $e_x(\psi)$ is a proper Levi subgroup of G. From the earlier remark on the independence of the conjugacy class of $e_x(\psi)$ on the choice of x it follows that this condition does not depend on the choice of x.

Fix an element $g_0 \in G$ in the conjugacy class in G defined by $e_x(\psi)$. Let $H_{g_0} \subset G$ be the centralizer of g_0 in G, which is a proper Levi subgroup.

Let

$$\psi: E \to E$$

be the automorphism in T(E) chosen above. So for any point $z \in E$, we have $\psi(z) = zg(z)$, where ψ as in (2.3) and

$$z \mapsto g(z) \in G$$

is a morphism of varieties from E to G.

Let

$$(2.4) \mathscr{S} \subset E$$

be the subvariety defined by all $z \in E$ with $g(z) = g_0$, where g_0 is the fixed element.

It is easy to see that for any point $x \in M$, the intersection $\mathcal{S} \cap E_x$ is nonempty and it is closed under the action of the centralizer H_{g_0} (here E_x denotes the fiber of E over x). Indeed, this is an immediate consequence of the identity

$$g(z\gamma) = \gamma^{-1}g(z)\gamma,$$

where $z \in E$ and $\gamma \in G$. Note that for each $z \in E$, the element g(z) is in the conjugacy class (in G) defined by $e_x(\psi)$ (which is independent of x).

From this identity it also follows that H_{g_0} acts transitively on the fibers of the projection of $\mathscr S$ to M. In other words, the subvariety $\mathscr S$ defines a reduction of structure group of the principal G-bundle E to the proper Levi subgroup H_{g_0} . This completes the proof of the proposition. \square

Remark 2.5. The above proposition justifies the "L" in L-indecomposability. An L-indecomposable G-bundle may admit a reduction of structure group to a proper reductive subgroup H of G with a maximal torus of H being a proper subgroup of a maximal torus of G. This happens for example when G is a torus and the G-bundle is trivial.

In the next section we will construct a reduction of the structure group of an L-decomposable G-bundle to a proper Levi subgroup H such that the principal H-bundle on M so obtained is L-indecomposable.

3. The Remak reduction

In this section G is a connected reductive linear algebraic group and E is a principal G-bundle over M.

As in (2.2), let T(E) be a maximal torus in $\operatorname{Aut}^0(E)$. The group T(E) gives a subgroup of G up to conjugation. To explain this, note that fixing a point $z \in E_x$, the group $\operatorname{Ad}(E)_x$ gets identified with G. Using this identification, the evaluation map $e_x : \operatorname{Aut}^0(E) \to \operatorname{Ad}(E)_x$ (that sends a section of $\operatorname{Ad}(E)$ to its evaluation at x) sends the subgroup $T(E) \subset \operatorname{Aut}^0(E)$ isomorphically to a subgroup of G. That the evaluation on T(E) is an isomorphism follows from the fact that the conjugacy class of $e_x(s)$, $s \in T(E)$, is independent of x—see proof of Proposition 2.4. Indeed, if the evaluation of $\psi \in T(E)$ at some point x is the identity map of the fiber E_x , then $\psi(y)$ is the identity map of E_y for all y.

Let $T_E \subset G$ be a subgroup in the conjugacy class defined by T(E). So T_E is contained in a maximal torus of G. Let H denote the centralizer of T_E in G. Note that H is a Levi subgroup of G. Clearly, H coincides with G if and only if T_E coincides with $Z_0(G)$.

Definition 3.1. With above notation the Levi subgroup H (which is determined up to inner conjugation) will be called the *Remak subgroup* for E.

It follows immediately from the above definition that the Remak subgroup for E coincides with G if and only if the principal G-bundle E is L-indecomposable.

Theorem 3.2. The principal G-bundle E admits a reduction of structure group to its Remak subgroup H. More precisely, once a maximal torus T(E) in $\operatorname{Aut}^0(E)$ is fixed, E admits a natural reduction to H. Furthermore, a principal H-bundle constructed this way from E is L-indecomposable.

Proof. Choose and fix a point $z_0 \in E_x$. Using z_0 , an isomorphism of $Ad(E)_x$ with G is obtained. Now, take T_E to be the image of T(E) in G by this isomorphism composed with the evaluation at x. Let

$$(3.1) f: T(E) \to T_E$$

be the resulting isomorphism (which depends on the initial choice of z_0).

Now, as in the proof of Proposition 2.4, set

$$\mathscr{S} \subset E$$

to be the subset defined by the property that for any $z \in \mathcal{S}$, the equality

$$\sigma(z) = zf(\sigma)$$

is valid for every $\sigma \in T(E)$. Here f is the map defined in (3.1); the map $\sigma : E \to E$ is the automorphism defined by $\sigma \in T(E)$.

It is straightforward to check that \mathscr{S} is closed under the action of the centralizer of T_E in G. In fact, \mathscr{S} defines a reduction of structure group of E to H.

Let E_H denote the principal H-bundle defined by the subvariety \mathcal{S} . Replace the fixed point z_0 by z_0g , where $g \in G$, but keep the maximal torus T(E) fixed. If

$$\beta: Ad(E)_x \to G$$

was the previous isomorphism for z_0 , then the new isomorphism for z_0g coincides with $Ad(g^{-1}) \circ \beta$, where $Ad(g^{-1})$ is the automorphism of G defined by

$$Ad(g^{-1})(h) = g^{-1}hg.$$

Therefore, T_E is replaced by

$$T_E' := \operatorname{Ad}(g^{-1})(T_E)$$

and H is replaced by $H' := \operatorname{Ad}(g^{-1})(H)$. This implies that $\mathscr S$ is replaced by $\mathscr S' := \mathscr Sg$.

If $F_Q \subset F$ is a reduction of structure group of a G-bundle F to a subgroup $Q \subset G$, then for any $g \in G$, the translation $F_Q g$ (of F_Q by g) is a reduction of the structure group of F to the subgroup $g^{-1}Qg$ of G. These two reductions are identified and we will not distinguish between them. Therefore, the reduction of structure group E_H of E to H defined by E_Q 0 coincides with the reduction of structure group of E1 to E2 to E3.

To prove that the above H-bundle E_H is L-indecomposable, first recall that $\operatorname{Aut}^0(E_H)$ is a subgroup of $\operatorname{Aut}^0(E)$. If E_H is L-decomposable, then a maximal torus of $\operatorname{Aut}^0(E_H)$ is larger (in dimension) than the center of H. In that case, a maximal torus of $\operatorname{Aut}^0(E)$ will be larger (in dimension) than the center of H, as $\operatorname{Aut}^0(E_H)$ is a subgroup of $\operatorname{Aut}^0(E)$. On the contrary, the maximal torus $T(E) \cong T_E$ is contained in the center of H. This is a contradiction; hence the H-bundle E_H must be L-indecomposable. This completes the proof of the theorem. \square

About the dependence of the reduction in Theorem 3.2 on the choice of the maximal torus T(E), we recall that any other maximal torus of $\operatorname{Aut}^0(E)$ is a conjugate of T(E). For any

$$\gamma \in \operatorname{Aut}^0(E)$$
,

consider the maximal torus

$$T'(E) := \gamma T(E) \gamma^{-1}$$

of $\operatorname{Aut}^0(E)$. For convenience replace the fixed point $z_0 \in E_x$ in the proof of Theorem 3.2 by the new fixed point $z_0' := \gamma(z_0)$.

For this new choice of maximal torus $\gamma T(E)\gamma^{-1}$ and z_0' , the torus $T_E \subset G$ remains unchanged (same as for T(E) and z_0), and hence the centralizer H is also unchanged. It is straightforward to check that $\mathscr S$ is replaced by $\mathscr S':=\gamma(\mathscr S)$. In other words,

$$E'_H := \gamma(E_H) \subset E$$

is the new reduction.

Therefore, if we replace the maximal torus T(E) by T'(E), then the new reduction differs from the earlier one by the automorphism γ of E.

For the dependence of the reduction in Theorem 3.2 on the choice of the torus T_E of G, replace T_E by $T'_E := g^{-1}T_E g$, with $g \in G$. Replace the fixed point z_0 by $z_0 g$ but keep T(E) fixed. Then, as we saw in the proof of Theorem 3.2, $\mathscr S$ gets replaced by $\mathscr S g$. Consequently, the new reduction is identified with the initial one (the identification is in the sense mentioned in the proof of Theorem 3.2).

Therefore, we have the following proposition:

Proposition 3.3. Up to an automorphism of E, Theorem 3.2 gives a unique reduction of structure group to a Levi subgroup.

Let G_1 (respectively, G_2) be a connected reductive linear algebraic group and E_{G_1} (respectively, E_{G_2}) a G_1 -bundle (respectively, G_2 -bundle) over M. Then the fiber product $E_{G_1} \times_M E_{G_2}$ is a principal $G_1 \times G_2$ -bundle over M. Let

$$E_{H_1} \subset E_{G_1}$$

(respectively, $E_{H_2} \subset E_{G_2}$) be the reduction of structure group obtained in Theorem 3.2. It is easy to see that the reduction of structure group

$$E_{H_1} \times_M E_{H_2} \subset E_{G_1} \times_M E_{G_2}$$

to $H_1 \times H_2$ coincides with the one given by Theorem 3.2 for $E_{G_1} \times_M E_{G_2}$.

Now we will prove a converse to Theorem 3.2. The converse says that if $E_L \subset E$ is a reduction of structure group of the G-bundle to a Levi subgroup L of G and the principal L-bundle E_L is L-indecomposable, then the reduction E_L coincides with the reduction obtained in Theorem 3.2 for some choice of the maximal torus T(E). We noted in Proposition 3.3 that choosing a different maximal torus T(E) of $\operatorname{Aut}^0(E)$ corresponds to changing the reduction of structure group by applying an automorphism of the E.

Theorem 3.4. Let L be a Levi subgroup of G and $E_L \subset E$ a reduction of structure group of a principal G-bundle to the subgroup L. If E_L is L-indecomposable, then there is a maximal torus T(E) of $\operatorname{Aut}^0(E)$ such that the reduction of structure group of E corresponding to T(E), obtained in Theorem 3.2, coincides with E_L .

Proof. Let $Z_0(L)$ denote the connected component of the center of the Levi subgroup L containing the identity element. So $Z_0(L)$ acts as automorphisms of the principal

L-bundle E_L . Since the L-bundle E_L is L-indecomposable, the group $Z_0(L)$ coincides with a maximal torus of $\operatorname{Aut}^0(E_L)$.

Now recall that $\operatorname{Aut}^0(E_L)$ is a subgroup of $\operatorname{Aut}^0(E)$. We will show that for any automorphism

(3.2)
$$\sigma \in \operatorname{Aut}^{0}(E) \backslash \operatorname{Aut}^{0}(E_{L})$$

in the complement, there is an element in $h \in Z_0(L) \subset \operatorname{Aut}^0(E)$ such that σ does not commute with h.

To prove this, take any point $y \in M$. If we fix a point in the fiber $(E_L)_y$ of E_L over y, then the fiber $\mathrm{Ad}(E_L)_y$ is identified with L as well as $\mathrm{Ad}(E)_y$ is identified with G (the trivialization of E_y is done using the image of y by the inclusion map of E_L in E). With these identifications, the above inclusion of $\mathrm{Ad}(E_L)_y$ in $\mathrm{Ad}(E)_y$ is simply the inclusion of L in G. The homomorphism of $Z_0(L)$ to $\mathrm{Ad}(E_L)$ coincides with the inclusion map of $Z_0(L)$ in L.

Note that for any automorphism σ as in (3.2), there exists a point $y \in M$ such that σ does not preserve the subvariety $(E_L)_y$ of E_y . On the other hand, the centralizer of $Z_0(L)$ in G coincides with L. Therefore, it follows immediately that there is an element $h \in Z_0(L)$ with the property that

$$(h\sigma)|_{E_v} \neq (\sigma h)|_{E_v}.$$

Consequently, $h\sigma \neq \sigma h$.

The above assertion that for any given element in $\operatorname{Aut}^0(E) \setminus \operatorname{Aut}^0(E_L)$ there is an element in $Z_0(L)$ not commuting with it immediately implies that $Z_0(L)$ is a maximal torus of $\operatorname{Aut}^0(E)$.

If we set T(E) in Theorem 3.2 to be this maximal torus $Z_0(L)$ and take the base point z_0 in the proof of Theorem 3.2 to be in E_L , then the reduction of structure group of E to the centralizer of $Z_0(L)$ in E_L 0 (which is E_L 1) constructed in Theorem 3.2 coincides with the reduction E_L 1. Indeed, this is an immediate consequence of the construction in Theorem 3.2. This completes the proof of the theorem. \square

Note that since a Levi subgroup of L, where L is a Levi subgroup of G, is also a Levi subgroup of G, an L-bundle E_L obtained by a reduction of structure group of a principal G-bundle E is L-indecomposable if E_L does not admit a further reduction to a Levi subgroup of G contained in L.

A reductive subgroup of G is called *irreducible* if it is not contained in a proper parabolic subgroup of G. A reducible subgroup is contained in the Levi subgroup of a proper parabolic subgroup of G. Therefore, for an L-indecomposable bundle E admitting a reduction of structure group to a reductive subgroup H, the inclusion of H in G is irreducible.

Remark 3.5. If $k = \mathbb{C}$ and M is a compact connected complex manifold, then all the results obtained in Sections 2 and 3 remain valid for holomorphic principal bundles (in

place of algebraic ones). Indeed, the only property of M that we have used is that the space of global functions on it are the constant ones.

Remark 3.6. Let M be a connected smooth projective curve over an algebraically closed field k of characteristic zero. An indecomposable vector bundle V over M admits a holomorphic connection if and only if $\operatorname{degree}(V) = 0$ [At2], p. 203, Proposition 19. Now, let E be an L-indecomposable G-bundle over M, where G is a connected reductive linear algebraic group. The G-bundle E admits a holomorphic connection if and only if for any character χ of G the associated line bundle $E_{\chi} := (E \times \mathbb{C})/G$ (the action of G on the right of \mathbb{C} is defined using χ) over M is of degree zero (see last paragraph of Section 1 (p. 335) of [AB]).

References

- [At1] M. F. Atiyah, On the Krull-Schmidt theorem with application to sheaves, Bull. Soc. Math. France 84 (1956), 307–317.
- [At2] M. F. Atiyah, Complex analytic connections in fibre bundles, Trans. Amer. Math. Soc. 85 (1957), 181–207.
- [AB] *H. Azad* and *I. Biswas*, On holomorphic principal bundles over a compact Riemann surface admitting a flat connection, Math. Ann. **322** (2002), 333–346.
- [Bo] A. Borel, Linear algebraic groups, Grad. Texts Math. 126, Springer-Verlag, New York-Berlin-Heidelberg
- [DM] F. Digne and J. Michel, Representations of finite groups of Lie type, London Math. Soc. Stud. Texts 21, Cambridge University Press, Cambridge 1991.
- [Gr] A. Grothendieck, A general theory of fibre spaces with structure sheaf, Lawrence, University of Kansas, Report no. 4, 1955.
- [St] R. Steinberg, Conjugacy classes in algebraic groups, Lect. Notes Math. 336, Springer-Verlag, New York-Berlin-Heidelberg 1974.

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Eingegangen 5. November 2002, in revidierter Fassung 12. Mai 2004