

Problem Set 2

Weighted Automata 2020

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Problem 1. Show that product of two semirings is a semiring, i.e. if S_1 and S_2 are semirings, then $S = S_1 \times S_2$, with the binary operators defined co-ordinatewise.

Problem 2. Is it possible to construct a weighted automata, over some semiring, for which the support language is non context free? Argue why. Use problem 1 as hint.

Problem 3. Construct a weighted automata \mathcal{A} over the semiring $(2^{\Sigma^*}, \cup, \cdot, \emptyset, \Sigma^*)$, such that

$$\begin{aligned} \llbracket \mathcal{A} \rrbracket : \Sigma^* &\rightarrow 2^{\Sigma^*} \\ w &\mapsto \{v \in \Sigma^* \mid v >_{lex} w\}, \end{aligned}$$

where $>_{lex}$ is the lexicographical ordering on the words. For example, $\epsilon <_{lex} a <_{lex} ab <_{lex} b$ and $\llbracket \mathcal{A} \rrbracket(a) = \Sigma^* \setminus \{\epsilon, a\}$.

Problem 4. Compute a basis of the reachable space of the following weighted automata on the rational semiring.

