- These problem sets are not graded. However students are strongly encouraged to solve these problems and submit solutions for feedback.
- Submissions shall be accepted till Friday, 7th February 2020 for feedback. Feel free to contact the TA in case of any doubts.

For the following questions, by a block in a word $\alpha \in \Sigma^\omega$, we mean a maximal continuous substring of identical letters.

1. Write an MSO formula $\varphi_0(x, y)$ such that $\varphi_0(x, y)$ holds iff x and y denote the beginning and end of a block of word α (both inclusive).
2. Write an MSO formula φ using φ_0 such that for a word $\alpha \in \{a, b\}^\omega$, $\alpha \models \varphi$ iff it satisfies the two conditions simultaneously:
   (a) α has infinitely many a's and b's; and
   (b) The parity of the length of the i^{th} block in w is equal to the parity of i.

In the following problems we will try to convert a Büchi automata to an equivalent MSO formula. Let $A = (Q, \Delta, q_1, F)$ be a Büchi automata with $Q = \{q_1, ..., q_k\}$. We will use the second order variables $X_1, ..., X_k$ to store the positions for which A would be in the states $q_1, ..., q_k$ respectively on a valid run of w.

3. Write an MSO formula $\varphi_1(X_1, ..., X_k)$ with k second-order free variables which is satisfied iff $X_1, ..., X_k$ form a partition of $\mathbb{N}$.
4. Write an MSO formula $\varphi_2(X_1, ..., X_k)$ which is satisfied iff the assignment of X_i's conforms to a valid run of the automata A.
5. Write an MSO formula $\varphi_3(X_1, ..., X_k)$ which is satisfied iff the assignment of X_i's conforms to the Büchi acceptance condition.
6. Write a formula φ using formulae $\varphi_1, \varphi_2, \varphi_3$, filling in the missing details such that $L(A) = L_\varphi$.
7. Make appropriate changes to your formula φ so that it works for finite words. In other words, for an NFA A, $L(A) = L_\varphi$.