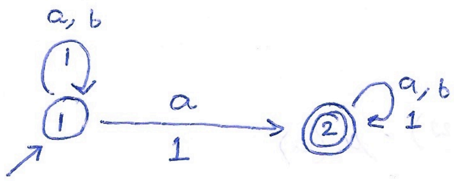
LECTURE - 1

Σ - finite alphabet

Language : $\Sigma^* \longrightarrow \{0, 1\}$

Weighted Language : $\Sigma^* \longrightarrow R$

weight domain



Transitions labelled by letter <u>and a weight</u>

What runs are there for a b b a?

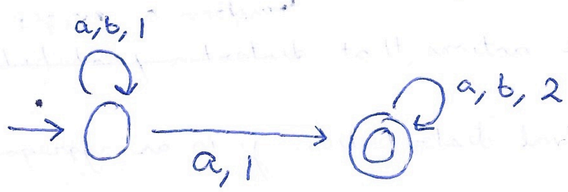
ρ_1 states: 1 1 1 1 2 $\longrightarrow$ 1

ρ_2 states 1 2 2 2 2 $\longrightarrow$ 1

} product of weights

Take sum of product of weights. So, $[\![A]\!] (a\,b\,b\,a) = 2$

The automata above counts the number of a's.



Convention : Not Mentioning an weight means 1.

What does this automaton compute?

Answer : the number represented by a, b in binary if a is 1, b is 0

The weights come from a semiring:

A *semiring* is a set S equipped with operations $+$ and $\times$ and distinguished elements 0 and 1 such that

- $(S, +, 0)$ is a commutative monoid
- $(S, \times, 1)$ is a monoid
- $\times$ distributes over $+$
- $a \cdot 0 = 0 \cdot a = 0$

This is often written as $(S, +, \times, 0, 1)$.

Caution: - Inverses may not exist
- $\times$ may not commute.

Examples

1. $(\mathbb{N}, +, \times, 0, 1)$
2. $(\mathbb{R}, +, \times, 0, 1)$
3. $(\mathbb{B}, \vee, \wedge, 0, 1)$ —— standard NFA are just weighted automata over this semiring
4. $(\{\infty\} \cup \mathbb{N}, \min, +, +\infty, 0)$ — tropical semiring
5. $(2^{\Sigma^*}, \cup, \circ, \{\}, \{\varepsilon\})$

We could also generalise initial and final states. Simply multiply an initial and a final weight at the states through which the run leaves and enters. (We use labelled arrows to show this)

Exercise Construct an automaton that computes the length of the minimal a block. Use the tropical semiring.

Simplifying Assumption: Assume your word is of the form $b \Sigma^* b$