# End-Semester Exam (ToC)

23/11/2016

*(Let $w^R$ denote the reverse of a word $w$. That is, if $w = a_1 a_2 \dots a_n$, then $w^R = a_n \dots a_1$.)*

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1. **[3 marks]** Consider the context-free grammar $G_1$ given below:

$$S \rightarrow aSb \mid aSbb \mid aaSb \mid aS \mid Sb \mid \epsilon$$

Is $L(G_1)$ regular ? If yes, give the *minimal* DFA. If not, argue using Myhil-Nerode theorem or pumping lemma.

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2. **[6 marks]** Consider the context-free grammar given by $G_2$ below.

$$\begin{aligned} S &\rightarrow aS \mid bA \mid aB \\ A &\rightarrow aS \mid bB \\ B &\rightarrow bS \mid aA \mid bA \mid \epsilon \end{aligned}$$

Is $L(G_2)$ regular? If yes, give the *minimal* DFA. If not, argue using Myhil-Nerode theorem or pumping lemma.

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3. **[9 marks]** For each of the following languages, state whether it is context-free or not. Justify your answers.

   (a) $\{xyx^Ry^R \mid x, y \in \{a,b\}^*\}$

   (b) $\{xyy^Rx^R \mid x, y \in \{a,b\}^*\}$

   (c) $\{xx^Rx \mid x \in \{a,b\}^*\}$

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The Intersection-non-emptiness problem of context-free grammars is given below.

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| Problem: | Intersection-Non-emptiness |
| Input: | $G_1$, $G_2$ : two context-free grammars |
| Question: | Is $L(G_1) \cap L(G_2) \neq \emptyset$? |

4. **[4 marks]** Show that Intersection-Non-emptiness is undecidable by a reduction from Post's correspondence problem.

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For a language $L \subseteq \Sigma^*$, let $\textsc{rev-closure}(L) = \{w \mid w \in L \text{ or } w^{\mathrm{R}} \in L\}$. Consider the following problem.

| | |
|---|---|
| Problem: | Reverse-closedness |
| Input: | a Turing Machine description $\langle M \rangle$ |
| Question: | Is $L(M) = \textsc{rev-closure}(L(M))$? |

5. **[5 marks]** Prove that Reverse-closedness is undecidable by a reduction from the halting problem for Turing machines.

6. **[5 marks]** Is the set $\{\langle M \rangle \mid L(M) = \textsc{rev-closure}(L(M))\}$ recursively enumerable? Is it co-recursively enumerable? Justify.

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