

Problem Set 3

Theory of Computation

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Don't Panic

Problem 1. Show that NFAs with only one final state are as powerful as the 'regular' NFAs. Is the same true for DFAs? Given any DFA, can we construct an equivalent DFA which contains only one final state?

Problem 2. For a given language $\mathcal{L} \subseteq \Sigma^*$, the reverse closure $\text{rcl}(\mathcal{L})$ is the minimal (under inclusion) subset of Σ^* containing \mathcal{L} such that for any word $w \in \text{rcl}(\mathcal{L})$, $w^r \in \text{rcl}(\mathcal{L})$. Show that such a subset exists, is unique, and is regular.

Problem 3. Suppose \mathcal{L} is a regular language. Let $\mathcal{L}^r = \{w \mid w \cdot w^r \in \mathcal{L}\}$ where w^r is the reverse of the string w and \cdot denotes concatenation. Show that \mathcal{L}^r is regular.

Problem 42. Let A and B be two sets. The difference between A and B , denoted by $A \setminus B$, is defined as $\{e \in A \mid e \notin B\}$. Symmetric difference of sets A and B is defined as $(A \setminus B) \cup (B \setminus A)$. Show that regularity is preserved under difference and symmetric difference.

Problem 5. Give an example of a regular language on a binary alphabet that cannot be recognized by a DFA with three states. Try to prove it formally.

So Long and Thanks For All the Fish