Problem Set 2

Theory of Computation

August 11, 2017

For the problem set is long and full of errors...

Problem 1. *Construct NFAs over the alphabet $\Sigma = \{a, b\}$ for the following languages and formally prove that the NFA accepts exactly the language defined.*

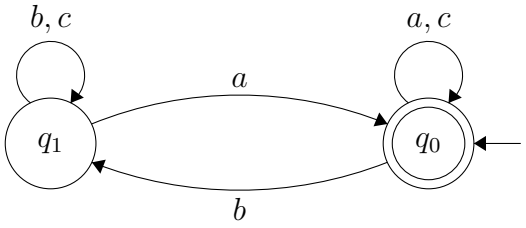
(1) $\mathcal{L}_1 = \{w \mid w$ *starts and ends with the same letter* $\}$

(2) $\mathcal{L}_2 = \{w \mid w$ *contains the substring* $aabb\}$

(3) $\mathcal{L}_3 = \{a, ab, abbab, bbbbb, aababa, bababaa\} \cup \{w \mid |w| > 8$ *and is even*$\}$

Try to convert the NFAs obtained for each of the above languages into DFAs.

Problem 2. *Consider the following automaton over the alphabet $\{a, c, b\}$. Is it a DFA?*



Try to identify the language it accepts, and formally prove the same.

Problem 3. *Do there exist two languages (over a non-empty alphabet) $\mathcal{L}_1$ and $\mathcal{L}_2$ such that $\mathcal{L}_1 \subset \mathcal{L}_2$ and $\mathcal{L}_2$ is regular but $\mathcal{L}_1$ is not? Can you show that for every language, there exists a regular (language) subset and a regular (language) superset of it?*

Problem 4. *If all words of size $\leq n$ are accepted by a complete DFA containing exactly n states, then what can you say about the language accepted by such a DFA? Does the answer remain the same for NFAs?*

Problem 5. *Construct an NFA that verifies addition of binary numbers. Suppose the problem is to add the numbers six and seven. then,*

$$\begin{array}{r} 0110 \\ +0111 \\ \hline 1101 \end{array}$$

We shall encode this as a string on the alphabet

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

where the first two rows represent the numbers to be added and the third row represents the sum. For instance, the above summation can be represented as:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Construct an NFA that takes a string on the alphabet $\Sigma = M_{3\times 1}(\{0,1\})$ *(the set of three cross one matrices with zeros and ones as entries), and accepts the string if it represents a valid instance of addition.*

How would you modify your automaton if the input was in decimal?

And now your problem set has ended.