SYLLABUS OF CORE COURSES FOR MSC MATHEMATICS

EFFECTIVE FROM 2025-26

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1. Graduate Algebra I

(1). Field theory: finite fields, separable and normal extensions, purely inseparable extensions, fundamental theorem of Galois theory. (2). Module theory, structure theorem for finitely generated modules over PIDs. (3). Multilinear algebra: tensor, symmetric and exterior products, tensor product of algebras. (4). Categories and functors, some notions of homological algebra.

References.

(1). S. Lang, *Algebra.*
(2). N. Jacobson, *Basic Algebra*, I and II.
(3). N. Bourbaki, *Algebra.*
(4). T. Hungerford, *Algebra.*
(5). M. Isaacs, *Algebra: A Graduate Course.*
(6). Additional sources as recommended by the instructor.

2. Graduate Analysis I

(1). Measure spaces, monotone class lemma, outer measures and Caratheodory extension, Lebesgue measure (invariant under translations), measurable functions, integration, MCT, Fatou's lemma, LDCT, product measures, Fubini. *Optional*: Convergence in measure, Lusin's theorem.

(2). Banach spaces, boundedness of linear operators, completeness of $B(X, Y)$, $C(X)$, dual space, Hahn-Banach (statement only proof later) , Heine-Borel, all norms are equivalent in finite dimensions, Hilbert spaces, Cauchy-Schwartz inequality, existence of orthonormal basis, Riesz lemma, Regularity of finite measures on

Date: 2025-08-01.

compact sets, $C(X)$ is dense in $L^p(X, \mu)$ for X compact and μ a finite measure. L^p spaces, completeness. *Optional*: Radon-Nikodym derivative, L^p-L^q duality.

(3). Basic Fourier analysis on the circle and on $\mathbb{R}$ (up to Plancheral theorem).

(4). Stone-Weierstrass theorem, Arzela-Ascoli theorem. *Optional*: Cantor set, complex measures, Riesz representation theorem.

References.

(1). Rudin, *Real and Complex Analysis.*

(2). Folland, *Real Analysis: Modern Techniques and Their Applications.*

(3). Stein and Shakarchi, *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*

(4). Torchinsky, *Real Variables.*

(5). Sunder and Athreya, *Measure and Probability.*

(6). Kesavan, *Functional Analysis*, TRIM 52.

3. Graduate Topology I

(1). Point-set topology: Connectedness and compactness, spaces, Tychonoff's theorem, separation axioms, normal and regular spaces, Urysohn-Tietze theorems, partitions of unity, compactifications.

(2). Fundamental groups and covering spaces, Van Kampen's theorem.

References.

(1). I. M. Singer and J. Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer

(2). A. Hatcher, *Algebraic Topology*, Cambridge University Press

(3). J. Munkres, *Topology*, Second Edition, Prentice Hall

4. Complex Analysis

Complex numbers and geometric representation, analytic functions, power series, exponential and logarithmic functions, conformality, Mobius transformations, complex integration, Cauchy's theorem, Cauchy's integral formula, singularities, Taylor's theorem, The maximum principle, The residue theorem and applications, Invariance of integrals under homotopy, Topology on space of holomorphic and meromorphic functions, Hadamard theory of entire functions, Order of entire functions, Picard's theorem, Automorphisms of Complex and Upper half plane, Analytic continuation, meromorphic continuation along a path, monodromy theorm, Riemann surfaces, branch points, analytic, meromorphic and holomorphic functions on Riemann surfaces.

References.

(1). D. Sarason, *Complex Function Theory*, 2nd ed. Hindustan Book Agency.

(2). G. A. Jones and D. Singerman, *Complex Functions*, Cambridge University Press.

(3). L. V. Ahlfors, *Complex analysis.* Tata Mc-Graw Hill.

5. Graduate Algebra II

Non-commutative rings, semisimple modules and rings, Jacobson theory, Artin-Wedderburn theorem, application to group-rings, introduction to finite group representations: Maschke's theorem, complex characters and orthogonality.

References.

(1). S. Lang, *Algebra.*
(2). N. Jacobson, *Basic Algebra*, I and II.
(3). N. Bourbaki, *Algebra.*
(4). T. Hungerford, *Algebra.*
(5). M. Isaacs, *Algebra: A Graduate Course.*
(6). Additional sources as recommended by the instructor.

6. Graduate Analysis II

(1). Hahn-Banach theorem,

(2). Baire's theorem and applications, open mapping, Closed graph, uniform boundedness principle (optional: applications to Fourier series).

(3). Weak, weak*-topology, Banach-Alaoglu theorem.

(4). Hilbert spaces: projections, unitaries, isometries, normal operators, spectral theorem for compact normal operators,

(5). *If time permits*: trace class, Hilbert-Schmid, partial isometries, Polar decomposition

(6). *optional*: One or more of the following topics: (a) Option I: Spectral theorem for normal operators, functional calculus, (b) Option II: Spectral theorem for unbounded self-adjoint operators. (c) Option III: Tempered distributions and the Fourier transform, applications to constant coefficient PDE.

References.

(1). Kesavan, *Functional Analysis*, TRIM 52.
(2). Taylor and Lay, *Introduction to Functional Analysis.*
(3). V. S. Sunder, *Operators on Hilbert space.*
(4). Paul R Halmos: *Introduction to Hilbert space and the theory of spectral multiplicity.*
(5). V. S. Sunder, *Functional analysis (Spectral theory)*

7. Graduate Topology II

(1). Recapitulation: Categories and functors. Chain and cochain complexes, (co)chain homotopy, homology of chain complexes. Exact sequences of chain complexes and the associated long exact sequence in homology. Universal coefficient theorem (proof may be omitted.)

(2). Simplicial/delta complexes, simplicial maps, barycentric subdivision, simplicial approximation theorem. Simplicial (co)homology and singular (co)homology properties-Eilenberg-Steenrod axioms. Mayer-Vietoris sequence. Relation between the fundamental group and H_1. Standard examples and applications (Homology Groups of spheres and projective spaces, compact surfaces, Brouwer fixed point theorem, invariance of domain, Borsuk-Ulam theorem.)

(3). One of the following topics: (a) cup and cap products, Künneth theorem, examples. (b) De Rham cohomology, statement of de Rham's theorem (if the students are familiar with differential forms and integration on manifolds)

(4). Notion of orientability of a manifold. Fundamental class of a (connected) manifold. Statement of Poincaré duality for manifolds and examples (proofs may be omitted).

References.

(1). A. Hatcher, *Introduction to Algebraic Topology*
(2). E. H. Spanier, *Algebraic Topology*
(3). F. Warner, *Introduction to differentiable manifolds and Lie groups*

8. Introduction to Manifolds

Recap of several variable calculus: open sets in $\mathbb{R}^n$, smooth functions, directional and partial derivatives, derivative of vector-valued functions as a linear map.

Definition and examples of smooth manifolds (Euclidean submanifolds and abstract manifolds, compatibility of atlases and the smooth structure, open sets, graphs of smooth functions, spheres, projective spaces, Grassmannians etc), and submanifolds. Smooth maps of manifolds, partial derivatives and the inverse function theorem. The tangent space at a point, the differential of a map, computing differential using curves, the chain rule, bases for the tangent space. Rank of a smooth map at a point, critical and regular points, the regular level set theorem, examples of regular submanifolds. The constant rank theorem, immersion and submersions. Bump functions and the partitions of unity. Vector fields - integral curves, local flows and Lie bracket. Differential forms and multilinear algebra. The exterior derivative. Orientation on manifolds, manifolds with boundary, integration and the statement of Stoke's theorem.

If time permits: additional material chosen by the instructor (E.g., Lie groups, Lie algebras and invariant forms on Lie groups.)

References.

(1). L. Tu, *An introduction to manifolds.*
(2). F. Warner, *Introduction to differentiable manifolds and Lie groups.*
(3). Additional sources as recommended by the instructor