NON-REGULAR ELECTIVES IN MATHS 2019-20

1. January-April term

(1) **Course Title:** Enumerative Combinatorics
Instructor: Anurag Singh

Topics: We will study some of the fundamental combinatorial structures that appear in most fields of mathematics. We will address the questions: Does an object with certain properties exist? If so, what structure does such an object have? How many such objects are there?

Objects: sets, permutations, posets, lattices, graphs, polytopes, etc. Methods: bijections, generating functions, Möbius inversion, algebraic and topological methods.

Textbooks: Richard Stanley. Enumerative Combinatorics, Vol. 1 (2nd edition).

(2) **Course Title:** Introduction to Modular forms
Instructor: Purusottam Rath

Outline: We will begin with modular forms for the full modular group, leading all the way to Hecke theory. Thereafter, we will introduce congruence subgroups and modular forms with characters. We will then introduce the L-series associated to these forms and derive the basic properties. Time permitting, we will introduce Hilbert modular forms and derive a result of Siegel, which is an extension of Euler's classical result on even zeta values to totally real number fields.

Textbooks: Ingham, Davenport, Tennenbaum

(3) **Course Title:** Stochastic Processes II
Instructor: B.V. Rao

1. Quick Review: Continuous random variables; conditional expectation. Martingales
2. Poisson Process: constructions using exponential RVs/using postulates on number of events axiomatic definition; order statistics, compound Poisson process, Poisson process with variable intensity. Application to Cramer-Lundberg Risk process in Insurance.
3. Continuous time Markov Chains: Kolmogorov differential equations, Birth-Death chains.
Application to Queuing models.

4. Discrete time continuous state chains.
Application to Random walk in convex sets.
5. Brownian motion. Elementary properties; peep into geometric Brownian motion; Gaussian processes with brief discussion of multivariate Gaussian distribution.
6. (If time permits) Glimpse into Markov Random fields and image models.

Some References:
1. S. Karlin and H.M. Taylor: A First Course in Stochastic Processes. Academic Press)
2. H M Taylor and S.Karlin: An introduction to stochastic modeling. (Academic Press)
3. Sheldon M Ross: Stochastic Processes; (John Wiley)
4. S. Ramasubramanian: Lectures on Insurance Models; (Hindustan Book Agency)
5. P G Hoel, S C Port, C J Stone: Introduction to Stochastic Processes; (Houghton Mifflin company.)
6.Pierre Bremaud: Markov Chains (Springer).

Prerequisites: SP1. Measure theoretic background would be helpful but not essential. Must be prepared to do Home Assignments.

(4) **Course Title:** Compact Riemann Surfaces.
Instructor: V. Balaji

Description: The course would aim to give complete proofs of the Riemann existence theorem, it's relationship with the inverse galois problem over function fields and to the last part of the course would be an attempt to prove the Uniformization theorem by the "method of continuity" following Klein and Poincare.

(5) **Course Title:** Completely positive and bounded maps
Instructor: Nirupama Mallick.

Outline of Topics: C*-algebra, positive elements, GNS construction, positive and completely positive maps, completely bounded maps, Russo-Dye theorem, Schwarz inequality, Schur maps, Choi-Kraus decomposition, Stinespring's dilation theorem, completely Positive Maps into Mn, Arveson's extension theorem and their completely bounded generalizations, Wittstock's extension theorem, structure theorem for completely bounded maps, Wittstock's decomposition theorem.

Prerequisites: Basic linear algebra and functional analysis.

References:
1) Positive definite matrices, Rajendra Bhatia, Princeton University Press, 2007
2) Completely bounded maps and operator algebras, V. I. Paulsen, Cambridge University Press, 2002

(6) **Course Title:** Étale cohomology and the Weil Conjectures
Instructor: Sukhendu Mehrotra

Outline: This course will cover the basic notions of étale cohomology necessary for understanding Deligne?s proof of the Weil Conjectures as in his article "Weil I." Some of the topics to be covered are étale sheaves, the proper base change theorem, Poincare duality, the Lefschetz trace formula. The course will closely follow Milne?s notes, especially its treatment of the proof of the Weil Conjectures.

(7) **Course Title:** Game Theory
Instructor: T. Parthasarathy

Rough Syllabus: Matrix games with examples. A constructive proof of minimal theorem using Linear Programming. Will discuss nonzerosum bimatrix games and the concept of Nash Equilibrium points. Lemke-Howson algorithm will be explained to construct an equilibrium Pair for bimatrix games. Several solution concepts will be introduced for cooperative games. Stochastic games will be mentioned and show how it differs from matrix games.

Textbooks: Students can refer to the recent lecture notes to be published soon written jointly With Dr Sujatha Babu. More or less all the topics mentioned above are discussed In these notes published by CMI. We will also mention several papers during the course Of the lectures.

(8) **Course Title:** Quantum Computation and Quantum Information.
Instructor: R. Srinivasan.

Outline of Topics: After introducing the frame work of quantum probability and proving uncertainity principle, we will discuss qubits, quantum gates, some simple circuits describing quantum teleportation and Deutch Algorithm. We will discuss more algorithms which includes Shor's algorithm. In the end we will discuss quantum error correcting codes(which includes (Knill-Laflamme Theorem) and some quantum information theory.

Prerequisites: Basic Linear algebra (more detailed linear algebra will be done in the course)

Basic reference:
1. TIFR Lecture notes of K. R. Parthasarathy available online
2. Quantum Information and Quantum Computation, Michael A. Nielsen & Isaac L. Chuang
3. Coding theorems of classical and quantum information theory, K. R. Parthasarathy