NON-REGULAR ELECTIVES IN MATHS 2019-20

1. Aug – Nov term

(1) **Course Title:** Algebraic Curves (ALGC)
Instructor: Biswajit Rajaguru

Description: The prerequisite for this course, is being comfortable with Commutative Algebra (CA), at the level of Atiyah-MacDonald. Specifically we will state the results from CA, we would need, but we will skip proving them in the class. We will follow the textbook (see below) closely and and try to reach chapter 8. A good advice to students interested in the course is to read chapter 1,2,4 of the textbook, **before the class starts**. It will acquaint them to the nature of objects in algebraic geometry. The textbook has lots of exercises. They are actually results one should know at that stage, and are used in the subsequent sections. So it is important to do these exercises.

Textbook: Algebraic Curves by Wiliam Fulton (can be downloaded legally from the web).

(2) **Course Title:** C^*-algebras and W^*-algebras (CAWA)
Instructor: Keshab Chandra Bakshi

Description: Gelfand-Naimark theory, Commutative C^*-algebras, Representations of C^*-algebras, The spectral theorem, Polar decomposition, Compact operators, The three locally convex topologies, the GNS representation, Geometry of projections, Preduals and W^*-algebras, Group von Neumann algebras, Group measure space construction, Crossed product algebras,von Neumann?s bicommutant theorem, Bounded Borel functional calculus, The Kaplansky density theorem, Normality and W^* algebras.

Prerequisites: Functional analysis, Measure theory, Complex analysis.

(3) **Course Title:** Coding Theory (CTH)
Instructor: Sharad Sane

Syllabus (tentative):
(a) Basic coding theory, Generator and Parity Check matrices, Maximum likelihood decoding and Shannon?s noisy channel theorem

(b) Some basic interesting codes and their properties, Hamming and Cyclic codes, Reed-Solomon codes, BCH codes, QR cdoes, Binary and Ternary Golay codes
(c) Weight enumerators and MacWilliams identities, Self-dual codes and their classficiation
(d) Bounds on codes, Gilbert-Varshamov bound, Hamming and Griesmer bounds, Orthogonal polynomials and linear programming bound
(e) Hadamard matrices, Plotkin bound and Levenshtein theorem
(f) Reed-Muller codes of higher orders and connections with Hadamard ma- trices of maximum excess and the Menon type Hadamard matrices
(g) Lloyd?s theorem on perfect codes
(h) Codes and designs: Assmus-Mattson theorem
(i) Lattices and codes

Textbooks:
(a) J.H. van Lint, An Introduction to Coding Theory, Springer Graduate Texts in Mathematics
(b) J. MacWilliams and N.J.A. Sloane, Theory of error correcting codes, North-Holland
(c) J. Birbrauer, An Introduction to coding theory, CRC Press

(4) **Course Title:** Geometric Group Theory (GGTH)
Instructor: Arpan Kabiraj

Description/Syllabus: The goal of the course is to cover the basics of Geometric group theory. The course will largely be self contained. The main focus will be the application of basic ideas to concrete examples. Topics covered will be as follows: Path metric spaces and group action on path metric spaces. Švarc-Milnor lemma, Cayley graphs, groups acting on trees and ping pong lemma. Quasi-isometry, hyperbolic groups, ends of groups and growth of groups. Basics of mapping class groups. The prerequisties are basic algebra, analysis and topology.
References:
(a) Translation (by W. E. Grossp) of the book by Ghys. Translation available online.
(b) *Office hours with a Geometric Group Theorist.* Edited by Matt Clay and Dan Margalit.
(c) *A primer on mapping class groups* by Benson Farb and Dan Margalit.

(5) **Course Title:** Introduction to Analytic Number Theory (ANT)
Instructor Biplab Paul

Syllabus: Introduction, Infinite product and series, Riemann-Stiltjes integrals, Abel summation/Partial summation formula. Distribution of primes: Euclid and Euler, asymptotic behaviour of $\pi(x)$, Riemann

ζ function, Euler product, Gamma function, Poisson summation formula, Functional equation and analytic continuation of $\zeta(s)$, zeros of $\zeta(s)$, estimates of non-trivial zeros of $\zeta(s)$, zero-free region and Riemann hypothesis, Explicit formula for $\psi(x)$, Prime number theorem with error term. Modular forms for $SL2(\mathbf{Z})$: Introduction, Modular group and modular functions, Eisenstein series and cusp forms, valance formula, space of modular forms, Fourier series expansion, estimates of coefficients of modular forms, Product expansion of ? function, Hecke operators, Hecke eigenforms and L-functions associated to them, Functional equation of L-function associated to eigenform and Hecke's converse theorem. If time permits, we shall give some applications.
Prerequisites: First course in complex analysis.

Textbooks:
(a) H. Davenport, Multiplicative number theory, Third edition, Revised and with a preface by Hugh L. Montgomery. Graduate Texts in Mathematics, 74, Springer-Verlag, New York, 2000.
(b) H. L. Montgomery and R. C. Vaughan, Multiplicative number theory. I. Classical theory, Cambridge Studies in Advanced Mathematics, 97, Cambridge University Press, Cam- bridge, 2007.
(c) J-P. Serre, A course in arithmetic, Translated from the French. Graduate Texts in Mathe- matics, No. 7, Springer-Verlag, New York-Heidelberg, 1973.
(d) G. Tenenbaum, Introduction to analytic and probabilistic number theory, Cambridge Studies in Advanced Mathematics, 46, Cambridge University Press, Cambridge, 1995.

(6) **Course Title:** Lie Algebras (LALG)
Instructor: Senthamarai Kannan

Syllabus/Desctiption Will follow *Introduction to Lie Algebras and Representation Theory* by James Humphreys.

(7) **Course Title:** Rigid Analytic Geometry (RAG)
Instructor: Pramathanath Sastry

Syllabus: This is an introduction to Tate's theory of rigid analytic spaces. The goal is to prove Mumford's Uniformization Theorem for rigid curves. Topics: Affinoid Spaces, Rigid Analytic Spaces, Rigid GAGA, Formal geometry, Uniformization of Rigid Analytic Curves.

Prerequisites: The students are expected to know schemes and cohomology of sheaves.
Textbooks:
(a) *Introduction to Rigid Geometry* byYichiao Tian.
(b) *Rigid Geometry of Curves and their Jacobians* by Werner Lütkebohmert.

(8) **Course Title:** Topics in Algebraic Surfaces II (AG2)
Instructor: Krishna Hanumanthu

Topics: The aim of the course is to introduce some current research problems on Seshadri constants and related topics. I will start with an introduction to Seshadri constants, the Nagata and SHGH Conjectures, and the bounded negativity conjecture. In the second half, I plan to discuss recent developments in the subject and introduce some open problems.

The course is a continuation of my course (Topics in Algebraic Surfaces - 1) in Jan-Apr 2019 semester. The contents of that course will serve as prerequisites. Roughly they are: divisors, differentials, and cohomology of sheaves on varieties; embeddings of projective varieties in projective spaces, positivity properties of line bundles such as ampleness, very ampleness; basics of curves and surfaces (specifically, Riemann-Roch theorems, intersection theory on surfaces, Hodge index theorem, blow ups).

References:
(a) Positivity in Algebraic Geometry I - Robert Lazarsfeld
(b) Global aspects of the geometry of surfaces - Brian Harbourne, available here: https://arxiv.org/pdf/0907.4151.pdf
(c) A primer on Seshadri constants - Thomas Bauer et al, available here: https://folk.uib.no/st00895/articolimiei/primer.pdf

(9) **Course Title:** Topological Data Analysis (TDA)
Instructor: Priyavrat Deshpande and Sourish Das

Background: The application of topological techniques to traditional data analysis, which before has mostly developed on a statistical setting, has opened up new opportunities. There is a growing interest to explore this field further as well as look for new applications. Some of the notable successes (such as the identification of a new type of breast cancer, or the classification of NBA players) in the recent years have earned praise from the industry.?Indeed, with the explosion in the amount and variety of available data, identifying, extracting and exploiting their underlying structure has become a problem of fundamental importance. Many such data come in the form of point clouds, sitting in potentially high-dimensional spaces, yet concentrated around low-dimensional geometric structures that need to be uncovered. The non-trivial topology of these structures is challenging for classical exploration techniques such as dimensionality reduction. The goal of TDA is therefore to develop novel methods that can reliably capture geometric or topological information (connectivity, loops, holes, curvature, etc) from the data without the need for an explicit mapping to lower-dimensional space.
Important note: This is not a course in algebraic topology. The objective of this course is to familiarize students with these new

methods, lying at the interface between pure mathematics, statistics and computer science.
Learning outcome: At the end of the course we expect that you will know how to obtain persistent bar codes for a given data set and interpret the answer to get new insight.

Syllabus:
Topology: Simplicial complexes, persistent homology, Betti numbers and bar codes.
Statistics: Dimensionality reduction, clustering, data reconstruction and inference techniques. A substantial part of the course will be devoted to hands on work like using various TDA packages, working through papers that describe applications of TDA to various data sets.
Project: For the project students will be assigned a research paper that deals with applications of TDA to a specific data data analysis problem. Their job will be to understand the paper, then implement the program and make sure that they can reproduce the results on the same data set and finally try and explore where else the idea of the paper works (this could be open ended).

Prerequisites: Linear algebra, topology of Euclidean spaces, basic statistics. Prior knowledge of algebraic topology is not needed, however, some maturity in point set topology would help.

Software Pakcages: Plex family (JavaPlex/jPlex)