ELECTIVE COURSES IN MATHS FOR JAN-APR 2023

Note: This list is subject to changes.

(1) **Algebra IV**
   (a) Instructor: Ramadas
   (b) Syllabus: Modules, generators and relations, structure theorem for Abelian groups/modules of Euclidean domains/PIDs. Applications to linear operators. Galois theory: separable and normal field extensions, fundamental theorem of Galois theory.
   (c) Textbooks:
      (i) Algebra by M. Artin
      (ii) Galois theory by E. Artin
      (iii) Algebra by S. Lang
   (d) Prerequisites: Algebra III
   (e) Target audience: BSc 3rd year students.

(2) **Stochastic processes II**
   (a) Instructor: S. Ramasubramanian
   (b) Syllabus:
      (i) Poisson Processes: Time-homogeneous Poisson process, waiting times, inter-arrival times, order-statistics property, non-homogeneous Poisson processes, Compound Poisson processes. Examples; applications.
      (ii) Continuous-time Markov chains: Introduction; holding times. Birth-death processes, backward and forward Kolmogorov equations. Limiting behaviour. Examples; applications.
      (iii) One-dimensional Brownian motion: Brief discussion; some elementary aspects, Brownian motion with drift. Transition probability density function.
   (c) Textbooks:
      (i) S. M. Ross: Stochastic Processes. 2nd edition.
      (ii) S. Karlin and H. M. Taylor: A first course in Stochastic Processes. 2nd edition.
      (iii) V. G. Kulkarni: Modelling and Analysis of Stochastic Systems. 2nd edition.
      (iv) H. C. Tijms: A first course in Stochastic Models.
   (d) Prerequisite: first course in probability and analysis. Basic knowledge of Markov chains is required (as was covered in the SP1 course).
   (e) Target audience: BSc 3rd year, MSc Maths/CS/DS.
   (f) Note: This course will be taught online.

(3) **Syzygies**
   (a) Instructor: Manoj Kummini
   (b) Syllabus: Regular sequences and depth, Koszul complexes, Cohen-Macaulay rings, Injective modules, Matlis duality, Canonical modules, Gorenstein rings, Local cohomology. Cohomology of coherent sheaves. Line bundles. syzygies of embeddings into projective spaces.
   (c) Textbooks:
      (i) The geometry of Syzygies by D. Eisenbud.
      (ii) Cohen-Macaulay Rings by W. Bruns and H. Herzog.
   (d) Prerequisite: Commutative algebra.
   (e) Target audience: BSc 3rd year, MSc Maths.

(4) **Algebraic Geometry II**
   (a) Instructors: Krishna Hanumanthu and Nabonita Ray
   (b) See `https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf` for the syllabus and a list of textbook(s).
   (c) Pre-requisites: Algebraic geometry 1.
   (d) Target audience: BSc 3rd year, MSc Maths

(5) **Coding Theory**
   (a) Instructor: Sharad Sane
   (b) Syllabus:
      (i) Basic coding theory, Generator and Parity Check matrices, Maximum likelihood decoding and Shannon's noisy channel theorem
      (ii) Some basic interesting codes and their properties, Hamming and Cyclic codes, Reed-Solomon codes, BCH codes, QR cdoes, Binary and Ternary Golay codes
      (iii) Weight enumerators and MacWilliams identities, Self-dual codes and their classficiation
      (iv) Bounds on codes, Gilbert-Varshamov bound, Hamming and Griesmer bounds, Orthogonal polynomials and linear programming bound
      (v) Hadamard matrices, Plotkin bound and Levenshtein theorem
      (vi) Reed-Muller codes of higher orders and connections with Hadamard matrices of maximum excess and the Menon type Hadamard matrices
      (vii) Lloyd's theorem on perfect codes
      (viii) Codes and designs: Assmus-Mattson theorem
      (ix) Lattices and codes
   (c) Textbooks:
      (i) J.H. van Lint, An Introduction to Coding Theory, Springer Graduate Texts in Mathematics
      (ii) J. MacWilliams and N.J.A. Sloane, Theory of error correcting codes, North-Holland
      (iii) J. Birbrauer, An Introduction to coding theory, CRC Press
   (d) Prerequisites:
   (e) Target audience: BSc 3rd year, MSc Maths

(6) **Lie groups**
   (a) Instructor: Parameswaran Sankaran
   (b) Syllabus:
      (i) Basic notions: Lie groups and their Lie algebras, exponential map, Lie subgroups, closed subgroups, examples. Compact Lie groups, maximal tori, structure of compact connected Lie groups. Classification of compact simple Lie groups (statement only). Structure of (non-compact) real reductive Lie groups–Cartan and Iwasawa decompositions.
      (ii) Haar measure, applications to representations of compact Lie groups. Peter-Weyl theorem, examples. Complex semi simple Lie groups and their representations. Weyl's unitarian trick.
      (iii) One or more topics from the following may be discussed in greater detail:
         - roots and weights, Weyl group (for compact/complex semi simple Lie groups).
         - Clifford algebra and the spin group.
         - Representations of compact connected Lie groups.
         - Cohomology: cohomology of Lie groups. Classifying spaces, homogenous spaces. Examples.
   (c) Textbooks:
      (i) Daniel Bump, "Lie groups", Springer.
      (ii) V. S. Varadarajan, Lie groups, Lie algebras and their representations, Springer.
      (iii) A. Knapp, Representation theory of semi simple Lie groups.
      (iv) J. Humphreys, Introduction to Lie algebras and their representations.
   (d) Prerequisites: The course will assume familiarity with basic concepts from topology, differential topology, (multi)linear algebra, covering spaces and fundamental groups.
   (e) Target audience: BSc 3rd year, MSc Maths

(7) **Quantum Gaussian states**
   (a) Instructor: Vasanth
   (b) Syllabus: The Weyl Operators and the Quantum Bochner's Theorem, the symplectic group, Gaussian States, covariant matrices, Williamson normal form, symmetric group of Gaussian states, Symplectic dilations, Gaussian channels. More topics if time permits.
   (c) Textbooks:
      (i) What is Gaussian state?, K. R. Parthasarathy Communications on Stochastic Analysis Vol. 4, No. 2 (2010) 143-160 (Survey article) (and first few papers of the author following this survey article)

    (ii) Gaussian states in continuous variable quantum information, Alessandro Ferraro, Stefano Olivares, Matteo G. A. Paris (Available at https://arxiv.org/pdf/quant-ph/0503237.pdf)
  (d) Prerequisites: The course Operators on Hilbert space and the frame work of quantum probability that was offered in Aug - Nov 2022.
  (e) Target audience: BSc 3rd year, MSc Maths

(8) **Partial differential equations**
  (a) Instructor: Mythily Ramaswamy
  (b) Syllabus:
    (i) Overview : Wellposed PDEs, Major examples of first and second order PDEs and some solution methods.
    (ii) First order PDEs : Solution by method of characteristics.
    (iii) Classification of second order PDEs, Solution by separation of variables and Fourier Series for Laplace, Heat and Wave equations.
    (iv) Laplace Equation : Fundamental Solution, Mean Value Properties, Maximum Principle, Poisson
    (v) Heat Equation : Fundamental Solution, Maximum Principle.
    (vi) Wave Equation : One dimensional wave equation and its solution, Higher dimensional wave equation.
  (c) Textbooks:
    (i) McOwen, PDE; Methods and Applications, Pearson.
    (ii) L.C. Evans, PDE, Graduate Studies in Mathematics, Vol 19, AMS.
    (iii) Qing Han, Basic course in PDE, Graduates studies in Mathematics, Vol 120, AMS
    (iv) Pinchover and Rubinstein, Introduction to PDE
  (d) Prerequisites: multivariable calculus and ODE theory.
  (e) Target audience: BSc 3rd year, MSc Maths
  (f) Note: This course will be taught online.

(9) **Introduction to sieve methods**
  (a) Instructor: Jyothsnaa Sivaraman
  (b) Syllabus:
    (i) Mertens' theorems
    (ii) Sieve of Eratosthenes
    (iii) Brun's sieve and applications
    (iv) Selberg's sieve
    (v) Brun-Titchmarsh theorem
    (vi) Large sieve inequality
    (vii) Statement of Bombieri-Vinogradov theorem and some applications.
  (c) Textbooks:
    (i) Sieve methods, by H. Halberstam and H. E. Richert.
    (ii) An introduction to sieve methods and their applications, by A.C. Cojocaru and M. Ram Murty.
  (d) Prerequisites: There are no specific prerequisites however some knowledge of elementary number theory will help in following the course.
  (e) Target audience: BSc 3rd year, MSc Maths

(10) **Groebner bases and applications**
  (a) Instructor: Selvaraja
  (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf. A course that is more application oriented, and does not assume much of the topics covered in the commutative algebra course. Polynomial rings and ideals, Operation on ideals, Monomial orders, Initial ideals, and Grobner bases. Buchberger's criterion, Buchberger's algorithm, reduced Grobner bases. Applications: Elimination of variables, Applications to operations on ideals, zero-dimensional ideals, The 3-color problem. Noetherian modules, module homomorphism, Monomial orders and initial modules, The division algorithm and Buchberger's criterion/algorithm for modules. syzygies, Schreyer's theorem.
  (c) Textbooks:
    (i) David Cox, John Little, and Donal O'Shea, Ideals, varieties, and algorithms.
    (ii) David Cox, John Little, and Donal O'Shea, Using Algebraic Geometry.

(d) Prerequisites: Algebra 3.
(e) Target audience: BSc 3rd year, MSc Maths.

(11) **Harmonic analysis**
(a) Instructor: Arghya Mondal
(b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf. The main aim is to cover locally compact abelian groups (Plancherel Theorem, Pontryagin duality and Fourier inversion formula) and compact groups (Peter Weyl Theorem).
(c) Textbooks: Principles of Harmonic analysis by Dietmar and Echterhoff
(d) Prerequisites: Topology, analysis and linear algebra.
(e) Target audience: BSc 3rd year, MSc Maths

(12) **Model theory**
(a) Instructor: Manoj Kummnini and S. P. Suresh
(b) Syllabus:
  (i) Basic model theory – compactness, completeness, Lowenheim-Skolem theorems, complete theories.
  (ii) Quantifier elimination – algebraically closed fields, real closed fields, applications.
  (iii) Realising and Omitting Types – Prime models, saturated models.
  (iv) Ramsey's theorem and indiscernibles.
(c) Textbooks:
  (i) Marker, Model theory.
  (ii) Poizat, A Course in Model theory.
(d) Prerequisites:
  (i) Knowledge of basic first-order logic. Logic course at CMI is recommended.
  (ii) basics of algebra: definitions and basic properties of groups, rings and fields
(e) Target audience: BSc 3rd year, MSc Maths/CS.

(13) **Introduction to intersection theory**
(a) Instructor: Barbara Fantecchi
(b) Syllabus: A reduced version (i.e., skipping some of the most technical proofs) in Fulton's Intersection Theory chapters 1 to 6, and if time allows give an outline of the GRR formula for morphisms of smooth projective varieties, focusing on statement and use rather than the proof.
(c) Textbooks: Intersection theory by W. Fulton
(d) Prerequisites: A first course in algebraic geometry
(e) Target audience: MSc and PhD Maths
(f) Duration: January 10th to March 10th (8 weeks, 2 lectures per wreek).

(14) **Stochastic Integration**
(a) Instructor: Rajeeva Karandikar
(b) Syllabus: The course will introduce continuous time stochastic processes, martingale theory needed for Stochastic integration, and move on to do Ito Integral and then cover Stochastic integration wrt Semimartingales.
(c) Textbook: Introduction to Stochastic Calculus by Rajeeva L. Karandikar and B. V. Rao (first four chapters)
(d) Prerequisites: Measure Theoretic Probability
(e) Target audience: BSc 3rd year.