ELECTIVE COURSES IN MATHS FOR AUG-NOV 2022

Note: This list is subject to changes.

(1) **Stochastic processes I**
   (a) Instructor: B V Rao
   (b) Syllabus: Random walks on integer lattices and graphs; Recurrence and Transience; stationary distribution and rate of approach; cover time, Examples – with excursions to general theory of Markov chains.
   (c) Textbooks:
      (i) G R Grimmett: Probability on graphs
      (ii) Yuval Peres etal: Markov Chains and Mixing times
      (iii) David Aldous etal: Reversible Markov Chains and Random walks on graphs
   (d) Prerequisite: first course in probability.
   (e) Target audience: BSc 3rd year, MSc Maths/CS/DS.

(2) **Commutative Algebra**
   (a) Instructors: Manoj Kummini and Animesh Lahiri
   (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
   (c) Target audience: BSc 3rd year, MSc Maths

(3) **Algebraic Geometry I**
   (a) Instructors: V Balaji and Amit Kumar Singh
   (b) Syllabus:
      (i) Pre-sheaves and sheaves of abelian groups (rings, algebras, modules over sheaves of algebras). Ringed spaces. Locally ringed spaces. Affine schemes. Schemes and quasi-coherent sheaves on them. Morphisms. Closed sub-schemes characterised by quasi-coherent ideal sheaves.
      (ii) Separated and proper maps.
      (iii) Elementary examples with curves and surfaces. Blowing up.
      (iv) Cohomology of sheaves. Derived functors, Čech cohomology.
      (v) Divisors, line bundles, linear systems.
      (vi) $\mathrm{Proj}(R)$ for a graded A-algebra R.
      (vii) Smooth varieites.
      (viii) Serre duality
      (ix) Curves, Riemann-Roch
      (x) Anything else, if time permits.
   (c) Pre-requisites and co-requisites:
      (i) *Commutative Algebra* The prime spectrum of a ring, flatness, Krull dimension, primary decomposition of modules, noether normalisation, nullstellensatz.
      (ii) *Homological algebra:* Derived functors on abelian categories via injective and projective resolutions, Ext^i, Tor_i.
   (d) Textbooks/References:
      (i) Hartshorne, *Algebraic Geometry*, GTM 52, Springer, New York, 1977.
      (ii) Kempf, *Algebraic Varieties*, LMS Lecture Notes Series 172, Cambridge University Press, Cambridge UK, 1993.
      (iii) Kempf, Some elementary proofs of basic theorems in cohomology of quasi-coherent sheaves, *Rocky Mountain Journal of Mathematics*, vol 10, Number 3, Summer 1990.
      (iv) Matsumura, *Commutative Ring theory*, Cambridge studies in advanced mathematics 8, Cambridge University Press, Cambridge, 1980.
      (v) The Stacks Project Authors, *Stacks Project*, https://stacks.math.columbia.edu, 2018
   (e) Target audience: BSc 3rd year, MSc Maths

(4) **Homological Algebra**
   (a) Instructor: Arvind Kumar

(b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
(c) Target audience: BSc 3rd year, MSc Maths

(5) **Optimization Techniques**
(a) Instructor: T Parthasarathy / Sujatha Babu
(b) Syllabus: The topics covered are as follows:
(i) Linear programming
(A) Introduction, formulation of a linear program (LP), definitions and theorems, Simplex algorithm + examples
(B) Duality (weak and strong duality theorem, Farkas lemma, complementary slackness theorem, KKT conditions for optimality)
(C) Transportation problem formulated as a LP (including theorems, revised simplex algorithm + example, unbalanced transportation problem, degeneracy)
(D) Linear Complementarity Problem (LCP)
(E) Solving large scale LPs using Dantzig-Wolfe decomposition technique
(ii) Nonlinear programming (NLP)
(A) Introduction to NLP
(B) 4 types of NLPs being considered
- Minimization problem (MP) - Local minimization problem (LMP) - Fritz John saddlepoint problem (FJSP) - Kuhn Tucker saddlepoint problem (KTSP)
(C) Connection between MP and LMP, MP and FJSP/KTSP
(D) Optimality conditions: Sufficiency Theorems + example to find optimal for FJSP; Constraint qualifications (CQ)- Slater's CQ, Karlin's CQ, Strict CQ; KTSP necessary optimality theorem + when there are linear equality constraints.
(iii) Dynamic programming
(A) Discounted Dynamic programming (DP)
(B) Discounted DP for finite state space: Operators $L(f)$ and T; Theorems leading to existence of optimal policies and (p, ϵ) optimal policies; Examples.
(C) Brief mention of existence of optimal policies relating to countable state space
(c) Prerequisites:
(i) Undergrad Real analysis
(ii) Undergrad algebra
(iii) Basic knowledge of Markov chains for the portion relating to Dynamic programming
(d) References:
(i) For LP, there are way too many excellent resources online and lots of books. No prescribed textbook.
(ii) For Non-linear programming: Chapter 5 from the book Non-linear Programming by Mangasarian
(iii) For Dynamic programming:
(A) Maitra's notes (ISI Calcutta): for topics relating to finite state space.
(B) D.Blackwell, Discrete dynamic programming(1962).
(C) D.Blackwell, Discounted dynamic programming(1965).
(e) Target audience: BSc 3rd year, MSc Maths

(6) **Coding Theory**
(a) Instructor: Sharad Sane
(b) Syllabus: (This is the syllabus from an earlier offering. Need to check.)
(i) Basic coding theory, Generator and Parity Check matrices, Maximum likelihood decoding and Shannon's noisy channel theorem
(ii) Some basic interesting codes and their properties, Hamming and Cyclic codes, Reed-Solomon codes, BCH codes, QR cdoes, Binary and Ternary Golay codes
(iii) Weight enumerators and MacWilliams identities, Self-dual codes and their classficiation
(iv) Bounds on codes, Gilbert-Varshamov bound, Hamming and Griesmer bounds, Orthogonal polynomials and linear programming bound
(v) Hadamard matrices, Plotkin bound and Levenshtein theorem
(vi) Reed-Muller codes of higher orders and connections with Hadamard matrices of maximum excess and the Menon type Hadamard matrices

   (vii) Lloyd's theorem on perfect codes
   (viii) Codes and designs: Assmus-Mattson theorem
   (ix) Lattices and codes
 (c) Textbooks:
   (i) J.H. van Lint, An Introduction to Coding Theory, Springer Graduate Texts in Mathematics
   (ii) J. MacWilliams and N.J.A. Sloane, Theory of error correcting codes, North-Holland
   (iii) J. Birbrauer, An Introduction to coding theory, CRC Press
 (d) Prerequisites:
 (e) Target audience: BSc 3rd year, MSc Maths

(7) **Algebraic Groups**
 (a) Instructor: S. Kannan
 (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
 (c) Target audience: BSc 3rd year, MSc Maths

(8) **Combinatorial and Geometric Group Theory**
 (a) Instructor: Oorna Mitra
 (b) Syllabus: Free groups, groups via generators and relations, free product and amalgamated free product, HNN-extension, normal forms of elements in these groups and some applications, some embedding theorems, trees and free groups with Schreier's theorem, Ping-Pong lemma, trees and amalgams, the structure of a group acting on a tree.
 (c) Textbooks: Trees by J.P Serre and Combinatorial group theory by Roger C. Lyndon, Paul E. Schupp
 (d) Prerequisites: Basic group theory and metric spaces
 (e) Target audience: BSc 3rd year, MSc maths.

(9) **Introduction to Differential geometry**
 (a) Instructor: Chaitanya Ambi
 (b) Syllabus: Riemannian metrics, Curves and Surfaces in the Euclidean 3-space, Affine connections, Curvature, Geodesics and parallel transport, Intrinsic geometry, Gauss-Bonnet Theorem (Theorema Egregium).
 (c) Textbooks: Differential Geometry -Connections, Curvature and Characteristic Classes by Loring Tu.
 (d) Prerequisites: Intro to Manifolds. A basic knowledge of Linear Algebra as well as Real Analysis will be assumed.
 (e) Target audience: MSc maths
 (f) Note: This is intended to be an introductory course in Differential Geometry. A special emphasis will be laid on curves and surfaces and problem solving throughout the course.

(10) **Enumerative combinatorics**
 (a) Instructor: Priyavrat Deshpande
 (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
 (c) Textbooks: Enumerative combinatorics vol 1 by R. Stanley and Hyperplane arrangements by R. Stanley
 (d) Prerequisites: Alegrba 3, discrete maths
 (e) Target audience: BSc 3rd year, MSc maths
 (f) Note: This is a 2 credit course and it will be offered after the mid-semester examination, i.e., in October-November 2022.

(11) **GIT and Moduli theory**
 (a) Instructor: Sukhendu Mehrotra
 (b) Syllabus: This course aims to cover the basic aspects of Geometric Invariant Theory as developed by Mumford, with the intention of applying them to construct some concrete moduli spaces. Moduli problems; algebraic group actions and quotients; affine GIT; linearlizations and projective GIT; stability criteria (topological, Hilbert-Mumford); the moduli of projective hypersurfaces; the moduli of vector bundles on a curve.
 (c) Textbooks:
   (i) "Moduli Problems and GIT" by V. Hoskins, available online here (the main reference).
   (ii) "Geometric Invariant Theory" by Mumford, Fogarty, Kirwan.
   (iii) "Introduction to Moduli Problems and Orbit Spaces" by Newstead.
   (iv) "The Moduli Space" by Halpern-Leistner, available at the author's webpage.

(d) Prerequisites: The course will also develop some of the algebraic geometry basics necessary for these applications. Knowledge of algebraic geometry at the level of Milne's notes or Kempf's book will be assumed (ie, varieties and cohomology of sheaves); exposure to schemes theory will be helpful but not absolutely necessary.
(e) Target audience: MSc/ PhD Math, BSc students with approval from their faculty advisor may also credit.

(12) **Operators on Hilbert space and the frame work of quantum probability**
(a) Instructor: Vasanth
(b) Syllabus: Operator theory on Hilbert spaces starting from the basics to spectral theorem for normal operator. This includes compact operators, trace class operators, Hilbert Schmidt operators etc. Theory of unbounded operators up to spectral theorem. Brief introduction to the framework of quantum probability. Observables in symmetric Fock spaces through Weyl representation and second quantization.
(c) Textbooks:
(i) Operators on Hilbert spaces, V. S. Sunder
(ii) Mathematical Foundation of Quantum Mechanics, K. R. Parthasarathy
(iii) Introduction to Quantum Stochastic Calculus, K. R. Parthasarathy
(d) Prerequisites: A basic analysis course (e.g. analysis III) and some knowledge of basic measure theory will be required. It can be learnt separately or in parallel by attending Graduate analysis I.
(e) Target audience: BSc 3rd year, MSc Maths.
(f) Note: This course serves as a prerequisite for *Quantum Gaussian States* being offered the following semester. However, the course on quantum Gaussian states is independent and not a continuation of this course.

(13) **Methods in Irrationality and Transcendence**
(a) Instructor: Siddhi Pathak
(b) Syllabus:
(i) Approximation of algebraic numbers: Liouville's theorem and generalizations.
(ii) Hermite's method: Transcendence of e.
(iii) Lindemann-Weierstrass theorem: Transcendence of $\alpha \in \overline{\mathbb{Q}} \setminus \{0\}$.
(iv) Hilbert's Seventh Problem: The Gel'fond Schneider theorem
(v) Baker's theory of linear forms in logarithm of algebraic numbers
(vi) E-functions and the Siegel-Shidlovskii method
(vii) Irrationality of values of the Riemann zeta-function
(c) Textbooks:
(i) Transcendental numbers by M. R. Murty and P. Rath
(ii) Transcendental numbers by N. I. Fel'dman and Yu. V. Nesterenko (Number Theory IV in the Encyclopedia of Mathematical Sciences Series)
(iii) Recent papers of J. Sprang, S. Fischler, W. Zudilin etc.
(d) Prerequisites: Algebra 2, Analysis 2, complex analysis.
(e) Target audience: BSc 3rd year, MSc Math

(14) **Elementary Number Theory**
(a) Instructor: R. Balasubramanian
(b) Syllabus: Farey series and Minkowski's theorem–irrational numbers–congruences–continued fractions–approximation by rationals–arithmetical functions.
(c) Textbook: G H Hardy and E M Wright, Introduction to number theory. Clarendon Press, 1975.
(d) Prerequisites: Apart from Mathematical maturity, we shall need familiarity with basic Riemann Integration, manipulation of sums (mostly finite), and Cauchy-Schwarz inequality.
(e) Target audience: Primarily a UG course, PG students can credit it with the consent of the faculty advisors.
(f) Note: This is supposed to be a first course in Number theory specially aimed at the uninitiated. The word elementary signifies that we shall not be using complex analytic tools. The contents of the book by Hardy and Wright constitutes the basic framework for our course, but this is not set in stone and we will proceed depending on the background and aptitude of the participants.

(15) **Algebraic Curves**
(a) Instructor: Amith Shastri

(b) Syllabus: Plane curves and their local properties, projective plane curves, Bezout's theorem, divisors and differntials the Riemann-Roch theorem and resolution of singularities for curves. This is essentially the entire textbook minus chapters 1, 2, 4.
(c) Textbook: Algebraic curves by W. Fulton.
(d) Prerequisites: Ring theory, familiarity with commutative algebra. Knowledge of covering spaces is an added bonus.
(e) Target audience: BSc 3rd year, MSc Maths.

(16) **Measure Theoretic Probability**
(a) Instructor: Rajeeva Karandikar
(b) Syllabus: Measure theory (short introduction) - introduction to Lebesgue integral, class of sets, sigma fields, monotone class theorem, measures on sigma fields, Caratheodary extension theorem (statement only), measurable functions and integration, Fatou's lemma, dominated convergence theorem, product of measure spaces and Fubini's theorem.
Probability theory - Kolmogorov's framework of probability theory, random variables and their distribution functions, independence, Kolmogorov consistency theorem, Sequence of independent random variables, Borel–Cantelli lemma, Kolmogorov 0-1 law, sums of independent random variables, strong law of large numbers, convergence in distribution, characteristic functions, central limit theorem for sequence of independent random variables.
(c) Suggested reading:
- P. Billingsley : Probability and Measure
- L. Breiman : Probability.
- W. Feller : An Introduction to Probability Theory and Its Applications, Volume 1
- K.L. Chung : Elementary Probability Theory.
- S.M. Ross : A First Course in Probability.
- R. Ash : Basic Probability Theory.
- P.G. Hoel, S.C. Port and C.J. Stone: Introduction to Probability Theory.
- J. Pitman : Probability.

(d) Prerequisites: Analysis and basic probability.
(e) Target audience: BSc 3rd year, MSc students should have consent of their faculty advisor.

(17) **Introduction to the BGG Category**
(a) Instructor: Upendra Kulkarni
(b) Syllabus: The goal is to understand the statement of the Kazhdan-Lusztig conjecture.
(i) The structure of complex semisimple Lie algebras and their classification in terms of root systems. This corresponds to chapters 1-3 (and part of 5) of the Humphreys Lie algebras book. (Approximately 1.5 months)
(ii) Basic study of the BGG category, corresponding roughly to chapters 1-4 of the second Humphreys book (on BGG category). This will in particular subsume the famous Weyl character formula for finite dimensional irreducible representations. (Approximately 1.5 months)
(iii) Introduction to relevant aspects of Coxeter groups: Bruhat order, Hecke algebra and construction of the KL polynomials. (Approximately 1 month)
(c) Prerequisites:
(i) Absolutely essential: thorough comfort with linear algebra and language of modules over noncommutative rings (Jordan decomposition, semisimplicity, Jordan-Holder)
(ii) Strongly desirable: exposure to basics of representations of groups and/or Lie algebras (Schur's lemma, tensor products/algebra, multilinear algebra of representations)
(iii) Desirable: comfort with objects defined by generators and relations (groups, modules, algebras), category theory language(universal properties, adjoint functors), basic homological algebra (projective modules, Ext).
(d) Textbook/ suggested reading: Three books by JE Humphreys: on Lie algebras,on BGG category and on Coxeter groups.
(e) Target audience: It is intended as a PhD/MSc elective, but motivated undergraduates are welcome provided they meet the prerequisites.
(f) Remarks:

(i) I will conduct (before and at the beginning of the semester) several sessions that will briskly develop the necessary prerequisites assuming only undergraduate algebra III. Students can attend these sessions as a standalone introduction to the prerequisites and to see whether the course is right for them.
(ii) There may be a follow-up course in the second semester. The goal of the second course (if it takes place) will be to understand the machinery that Elias and Williamson used to complete Soergel's program to provide a purely algebraic proof of the KL conjecture. This machinery creates a kind of "Hodge theory" in a purely algebraic situation, even when there is no apparent underlying geometry that explains it. Similar amazing phenomena are being discovered in combinatorics, for which June Huh just won a Fields medal.